8 Hour

Chapter-5 (Continuity and Differentiability)

Limit

Limit : Limit is a common expected value of the function at a point and Point $x=a \in R$ from both the side

* Do not use L-opital Rule and again do not use $\lim_{x\to 0} \frac{1-\cos x}{x^2} = \frac{1}{2}$ do not directly.

$$RHL = \lim_{x\to a^+} f(x)$$

$$LHL = \lim_{x\to a^-} f(x)$$

RHL = LHL = Existence of Limit at point $x = a$

$$RHL = LHL = \lim_{x\to a} f(x)$$

① **Direct Subs :**

eg. $\lim_{x\to 1} \frac{x^2-1}{x^5+1} = \frac{1-1}{1+1} = \frac{0}{2} = 0$

② **Facto :**

$$\lim_{x\to 1} \frac{x^3-1}{x-1} \left(\frac{0}{0}\right)$$

$$\lim_{x\to 1} \frac{(x-1)(x^2+x+1)}{(x-1)}$$

$$\lim_{x\to 1} (x^2+x+1) = 3$$

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(3) Rationali.

$$\lim_{x\to 0} \frac{\sqrt{1-x}-1}{x} \quad \left(\frac{0}{0}\right)$$

$$\lim_{x\to 0} \frac{(\sqrt{1-x}-1)\times(\sqrt{1-x}+1)}{x[(\sqrt{1-x}+1)}$$

(4) Trigo.

use only

$$\lim_{x\to 0} \frac{\sin x}{x} = 1 = \lim_{x\to 0} \frac{\tan x}{x}$$

Eg: $\lim_{x\to 0} \frac{(1-\cos x)}{x^2} \left(\frac{0}{0}\right)$ $\left(\cos x = 1-2\sin^2\frac{x}{2}\right)$

$$\Rightarrow \lim_{x\to 0} \frac{(2\sin^2 x/2)}{x^2}$$

$$= 2 \lim_{x\to 0} \left(\frac{\sin x/2}{2\,x/2}\right)^2$$

$$= \frac{2}{4} = \frac{1}{2}$$

5) Substitution Method:

$$\lim_{x\to a} \frac{f(x)}{g(x)} \left(\frac{0}{0}\right) \qquad a \neq 0$$

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Put $x = a+h$, $h \to 0$

$$\lim_{h \to 0} \frac{f(a+h)}{g(a+h)}$$

Ex $\lim_{x \to \pi/2} \frac{1-\cos x}{(\pi-2x)^2}$ $\left(\frac{0}{0}\right)$ - ∞

Put $x = \frac{\pi}{2}$

$$\lim_{x \to \pi/2} = \frac{1-0}{\pi - \pi/2} = \frac{1}{0} \to \infty$$

$x \to \left(\frac{\pi}{2}+h\right)$ $h \to 0$

Q: $\lim_{x \to \pi/2} (\sec x - \tan x)$

$$\lim_{x \to \frac{\pi}{2}} \left(\frac{1}{\cos x} - \frac{\sin x}{\cos x}\right) = \lim_{x \to \frac{\pi}{2}} \left(\frac{1-\sin x}{\cos x}\right)$$

Put $x = \frac{\pi}{2} + 4$

$$\lim_{h \to 0} \left[\frac{1-\sin\left(\frac{\pi}{2}+h\right)}{\cos\left(\frac{\pi}{2}+h\right)}\right] = \lim_{h \to 0} \left[\frac{1-\cos h}{-\sin h}\right]$$

$$= \lim_{h \to 0} \left[\frac{2\sin^2 h/2}{-2\sin\frac{h}{2}\cos\frac{h}{2}}\right]$$

$$= \lim_{h \to 0} \left[\frac{\sin h/2}{-\cos h/2}\right] = 0$$ Ans

R R Lalwani

① Class work
② Misc.
③ NCERT

Ex = 5.1

Continuity : A function $f(x)$ be continuous at a point $x = a \in R$

if RHL = LHL = Value $f(a)$

$$\left\{ \begin{array}{l} RHL = \lim\limits_{x \to a+} f(x) \\ \\ LHL = \lim\limits_{x \to a^-} f(x) \end{array} \right\}$$

Q. Show that ~~modulus fn~~ of ~~f(x) ∈ R~~

Type - 1

Q.
$$f(x) = \begin{cases} g(x) : x > a & (RLH) \\ h(x) : x = a & (\text{Value of } f(x)) \\ l(x) : x < a & (LHL) \end{cases}$$

Type - 2 :

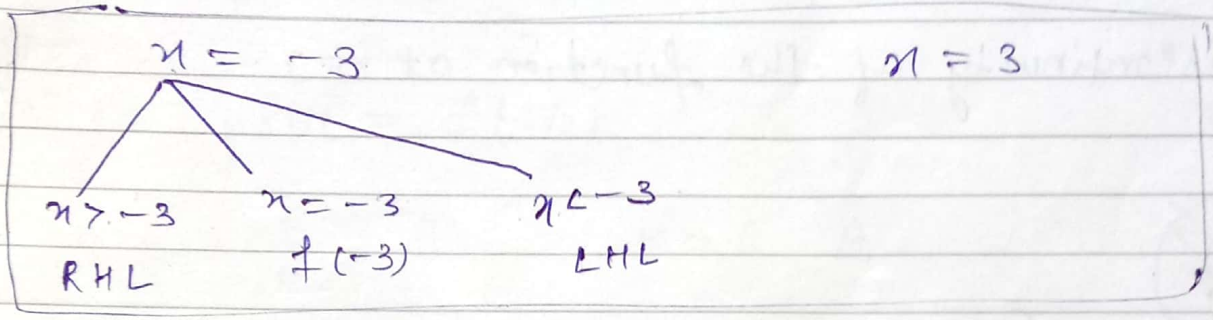
$$f(x) = \begin{cases} g(x) : x \neq a & (\text{LHL or RHL}) \\ h(x) : x = a & (\text{Value of } f(x)) \end{cases}$$

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NCERT Ex 5.1

Q.7 ⇒ $$f(x) = \begin{cases} |x| + 3 & ; \quad x \leq -3 \\ -2x & ; \quad -3 < x < 3 \\ 6x + 2 & ; \quad x \geq 3 \end{cases}$$

Find all the Point of discontinuity of $f(x)$.



The Possible points of this Continuity are
$x = -3, 3$

(1) at $x = -3$

$$RHL = \lim_{x \to a^+} f(x)$$

$$\Rightarrow \lim_{x \to -3^+} (-2x) \Rightarrow (-2)(-3) = 6$$

$$LHL = \lim_{x \to a^-} f(x)$$

$$\Rightarrow \lim_{x \to -3^-} |x| + 3 \Rightarrow |-3| + 3 = 6$$

$f(x) = |x| + 3$

$f(-3) = |-3| + 3 = 6$

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(ii) at $x=3$

$$LHL = \lim_{x \to 3^-} f(x) \quad \lim_{x \to 3^-} (-2x) = -2 \times 3 = -6$$

discontinuity of the function at $x=3$

Q. 26

$$f(x) = \begin{cases} \frac{k \cos x}{\pi - 2x} & ; \quad x \neq \pi/2 \\ 3 & ; \quad x = \pi/2 \end{cases}$$

Find the value of k

$$RHL = LHL = f(\pi/2)$$

$$\lim_{x \to \pi/2} f(x) = 3 \quad \text{or}$$

$$RHL = \lim_{x \to \pi/2^+} \frac{k \cos x}{\pi - 2x} \quad \left(\frac{0}{0}\right)$$

Put $x = \frac{\pi}{2} + h \quad h \to 0$

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$$\Rightarrow \lim_{h\to 0} \frac{K\cos\left(\frac{\pi}{2}+h\right)}{\pi-2\left(\frac{\pi}{2}+h\right)}$$

$$\Rightarrow \lim_{h\to 0} \frac{+K\sin h}{+2h}$$

$$\frac{K}{2}\lim_{h\to 0}\frac{\sin h}{h} = \frac{K}{2}$$

So $RHL = f(\pi/2)$

$$\frac{K}{2} = 3 \quad , \quad K = 6 \quad \text{Ans.}$$

Q. $f(x) = \begin{cases} \dfrac{\sin(a+1)x + 2\sin x}{x} & ; x<0 \\ 2 & ; x=0 \\ \dfrac{\sqrt{1+bx}-1}{x} & ; x>0 \end{cases}$ at $x=0$

Find the value of a and b if f^n is Cont. at $x=0$

f^n is continuous at $x=0$

So, $LHL = RHL = f(0)$

$$LHL = \lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{\sin(a+1)x + 2\sin x}{x} \left(\frac{0}{0}\right)$$

$$\lim_{x\to 0^-}\left[(a+1)\frac{\sin(a+1)x}{(a+1)x} + 2\frac{\sin x}{x}\right] =$$

$$= (a+1)+2 = a+3$$

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$$RHL = \lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{\sqrt{1-bx}-1}{x}$$

$$\lim_{x\to 0^+} \frac{\sqrt{1-bx}-1}{x} \times \frac{\sqrt{1-bx}+1}{\sqrt{1-bx}+1} = \frac{1-bx-1}{x(\sqrt{1-bx}+1)}$$

$$= \lim_{x\to 0^+} \frac{-bx}{x(\sqrt{1-bx}+1)} = \frac{-b}{2}.$$

$$a+3 = \frac{-b}{2} = 2 \qquad a = -1$$
$$b = -4$$

Q.2 $$f(x) = \begin{cases} \frac{1-\cos kx}{x \sin x}, & x \neq 0 \\ 1/2 : & x = 0. \end{cases}$$ at $x=0$ contin. find k.

f^n is continuous at $x=0$

So, LHL = RHL = $f(0)$ (Given)

$$LHL \Rightarrow \lim_{x\to 0^-}$$

$$\lim_{x\to 0} \frac{1-\cos kx}{x^2\left(\frac{\sin x}{x}\right)}$$

$$\lim_{x\to 0} \frac{1-\cos kx}{x^2}$$

$$\lim_{x\to 0} \frac{2\sin^2 kx/2}{x^2}$$

$$2 \lim_{x\to 0} \left(\frac{k/2 \sin kx/2}{\frac{k}{2}x}\right)^2$$

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$$\frac{k^2}{2} \times 1 = \frac{k^2}{2}$$

$$\frac{k^2}{2} = \frac{1}{2}$$

$$k = \pm 1$$

3|27
9|1

2017

Qu: $$f(x) = \begin{cases} \dfrac{(x+3)^2 - 36}{x-3} & ; \ x \neq 3 \\ k & ; \ x = 3 \end{cases}$$

$$\lim_{x \to 3} \frac{(x+3+6)(x+3-6)}{(x-3)}$$

$$k = 12$$

$(x+3)^2 - 36$

$: x^2 + 9 + 2 \cdot 3 \cdot x - 36$

$x^2 + 6x - 27$

$x^2 + 3x + 9x - 27$

$x(x+3) + 9(x+3)$

$(x+3)(x+9)$

Q. $$f(x) = \begin{cases} \dfrac{e^{1/x} - 1}{e^{1/x} + 1} & ; \ x \neq 0 \\ 0 & ; \ x = 0 \end{cases}$$

Show that this fn is disf. at $x = 0$

LHL $\neq$ RHL

$$LHL = \lim_{x \to 0^-} \left(\frac{e^{1/x} - 1}{e^{1/x} + 1} \right)$$

$$RHL = \lim_{x \to 0} f(0+h) \Rightarrow \lim_{h \to 0} \frac{e^{1/h} - 1}{e^{1/h} + 1}$$

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$e^{-\infty} = 0$

$e^{\frac{1}{0}} = 0$ ∴

$\lim_{h \to 0} \frac{1 - e^{-1/h}}{1 + e^{-1/h}}$

$\frac{e^{1/n}}{e^{1/n}}$

$\frac{1 - e^{-\infty}}{1 - e^{-\infty}} = \frac{1 - 0}{1 + 0} = 1$

Que: $f(x) = \begin{cases} \frac{1 - \cos x \sqrt{\cos 2x}}{x^2} & ; x \neq 0 \\ k & ; x = 0 \end{cases}$

$k = \frac{3}{2}$

Find k

$\lim_{x \to 0} \frac{1 - \cos x \sqrt{\cos 2x}}{x^2}$

$= \frac{1}{x^2} - \frac{\cos x \sqrt{\cos 2x}}{x^2}$

$= \frac{1}{x} \times \frac{\sqrt{2}}{1}$

$1 - \cos x \sqrt{2\cos^2 x - 1}$

$\frac{1}{x^2}$

$\frac{0}{0}$

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Differenciability:

A function $f(x)$ be differenciable at a point $x = a \in R$ iff

$$RHD = LHD$$

$$RHD = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$LHD = \lim_{h \to 0} \frac{f(a-h) - f(a)}{-h}$$

D.59

e.g. Show that at $x=1$ f not deriavitive.

$$f(x) = |x-1| \quad x=1$$

$$f(x) = \begin{cases} (x-1) & x \geq 1 \\ -(x-1) & x < 1 \end{cases}$$

$$RHD = \lim_{h \to 1} \frac{f(1+h) - f(a)}{h}$$

$$= \lim_{h \to 1} \frac{(1+h-1) - (1-1)}{h}$$

$$= \lim_{h \to 1} \frac{h-0}{h} = 1$$

$$RHD = \lim_{h \to 1}$$

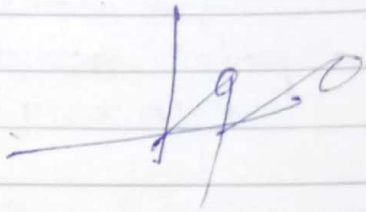
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Q.10 Show that $[x]$ is not diff. at integers at $x=1, 2$

L.H.F

$[a+h] = a$

$[a-h] = a-1$



$$RHD = \lim_{a \to 1} \frac{f(a+h) - f(a)}{h}$$

$$= \lim_{h \to 0} \frac{[1+h] - [1]}{h}$$

$$= \lim_{h \to 0} \frac{1-1}{h} = \lim_{h \to 0} \frac{0}{h} = 0.$$

LHD =

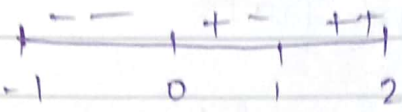
$$= \frac{0+1}{+h} = \frac{1}{0} = \infty$$

$RHD \neq LHD$

Q. Show that $f(x) = |x| + |x-1|$ diss. the continuity and diff. at $(-1, 2)$

$$f(x) = \begin{cases} 1-2x & -1 < x \leq 0 \\ 1 & 0 < x \leq 1 \\ 2x-1 & 1 < x < 2 \end{cases}$$

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at $x = 0$

$$RHD = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h \to 0} \frac{|1|}{1} = \lim_{h \to}$$

$$LHD = \lim_{x \to 0} \frac{f(a-h) - f(a)}{-h}$$

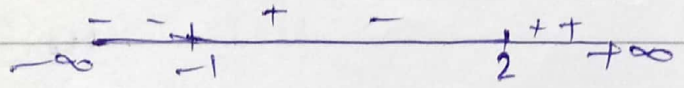
$$= \lim_{h \to 0} \frac{1 - 2x}{}$$

$$= \lim_{h \to 0} \frac{1 - 2(a-h) - f(0)}{-h}$$

$$= \lim_{h \to 0} \frac{1 + 2h - 1}{-h}$$

Q. $f(x) = |x+1| - |x-2|$ diff

Show that the given f^n not diff at $x = ?$



$$f(x) = \begin{cases} -(\quad) = (x-2) & x \leq -1 \\ +(x+1) + (x-2) & -1 \leq x < 2 \\ & x \geq 2 \end{cases}$$

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$$RHD = \lim_{a \to -1} \frac{f(a+h) - f(a)}{h}$$

$$= \lim_{h \to 0} \frac{f(-1+0h) - f(-1)}{h}$$

$=$

$\cos 2n = 2\cos^2 n - 1$

$\sin 2n = 1 - 2\sin^2 n$

Differenciation

* Diff :

1st Principle of Differenciation

$$\frac{dy}{dx} = ?$$

$$y = \sin x$$

$$\boxed{\frac{dy}{dx} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}}$$

(1) Trigon :— (i) $1 + \cos 2x = 2\cos^2 x$

(ii) $1 - \cos 2x = 2\sin^2 x$

(iii) $\sqrt{1 + \sin 2x} = \sqrt{(\cos x + \sin x)^2}$

(iv) $= |\cos x + \sin x| = \pm$

(v) $\sqrt{1 - \sin 2x} = \sqrt{(\cos x - \sin x)^2} = |\cos x - \sin x| = \pm$

(5) $\frac{1 \pm \tan x}{1 \mp \tan x} = \tan\left(\frac{\pi}{4} \pm x\right)$

(2) Basic diff :

$$\ln x = 1/x$$

$$\log_e |x| = 1/x$$

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$x^n = u$

$\frac{d}{dx}\left(\frac{1}{x}\right) = \frac{-1}{x^2}$

$\frac{d}{dx}\left(\frac{1}{x^2}\right) = \frac{-2}{x^3}$

$\frac{1}{x^{100}} = \frac{-100}{x^{101}}$

$\frac{d}{dx}\left(\log_{10} x\right) = \frac{d}{dx}\left(\frac{\log_e x}{\log_e 10}\right) =$

$= \log_e x \frac{d}{dx}\left(\frac{1}{\log_e 10}\right) =$

* Fundamental Rules of Differencation:

① $\frac{dy}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}$

② $\frac{d}{dx}(u \cdot v) = u \cdot v' + v \cdot u'$

③ $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \cdot u' - u \cdot v'}{v^2}$

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④ Chain Rule [Diff. of Composite fⁿ] :-

Chain	Product (v.u)
$y = fog(x)$	$y = f(x).g(x)$

① $y = \sin^2 x$ ② $y = \sin x^2$ ③ $y = \sin^2 x^2$

$y = (\sin x)^2$ $\frac{dy}{dx} = \cos x^2 \times 2x$ $\frac{dy}{dx} = 2\sin x^2 \cos x^2 \times 2x.$

$\frac{dy}{dx} = 2\sin x \cos x$

$= \sin 2x$

Q. If $y = \log\left[\tan\left[\frac{\pi}{4} + \frac{x}{2}\right]\right]$ then prove that $\frac{dy}{dx} = \sec x$

$\log\left[\tan\left[\frac{\pi}{4} + \frac{x}{2}\right]\right]$

$\frac{dy}{dx} = \frac{1}{\tan(\frac{\pi}{4} + x/2)} \times \sec^2(\pi/4 + x/2) \times \left[0 + \frac{1}{2}\right]$

$\frac{dy}{dx} = \frac{1}{2\sin(\frac{\pi}{4} + \frac{x}{2})\cos(\frac{\pi}{4} + \frac{x}{2})} = \frac{1}{\sin 2(\frac{\pi}{4} + \frac{x}{2})} = \frac{1}{\sin(\frac{\pi}{2} + x)}$

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$$\frac{1}{\cos x} = \sec x$$

Q. $y = \log\left[x + \sqrt{a^2+x^2}\right]$ $(a^2+x^2)^{1/2}$ x^2

$$\frac{dy}{dx} = \frac{1}{\left[x+\sqrt{a^2+x^2}\right]} \times \left[1 + \frac{1}{2\sqrt{a^2+x^2}} \times 2x\right]$$

$$= \frac{1}{x+\sqrt{a^2+x^2}} \times \frac{\sqrt{a^2+x^2}+x}{\sqrt{a^2+x^2}}$$

Q. $y = \log\sqrt{\frac{1+\sin x}{1-\sin x}}$

Method ⇒ 1. Chain Rule

(2) Log Property

(3) Perfect square (4) trigo – Sin → Cos. $(90-\theta)$

Best (5) Rationalisation.

$\tan\frac{\pi}{4}+\theta$

$$y = \log\sqrt{\frac{1+\sin x}{1-\sin x} \times \frac{(1+\sin x)}{(1+\sin x)}}$$

$$= \log\left(\frac{1+\cos x}{\cos x}\right)$$

$$y = \log(\sec x + \tan x)$$

$$\frac{dy}{dx} = \frac{1}{\sec x + \tan x} \times (\sec x \tan x + \sec^2 x)$$

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Class-work
Board - Booklet = Miscellet
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~~Exemplar~~/Illustration
Do Yourself-Q.

$y = \sin^{-1}[2x\sqrt{1-x^2}]$ Put $x = \sin x$

5.3

Method of differentiation:

* Differentiation of Implicit function:

↓

[Do not express y as a function of x]

$y \xrightarrow{x} \frac{dy}{dx}$ $\quad y^2 \xrightarrow{x} 2y \times \frac{dy}{dx}$

$\frac{1}{y} \xrightarrow{x} -\frac{1}{y^2} \cdot \frac{dy}{dx}$ $\quad \log y \xrightarrow{x} \frac{1}{y} \times \frac{dy}{dx}$

* $\frac{d}{dx}(x \cdot y) = x\frac{dy}{dx} + y \cdot 1$

$\frac{d}{dx}\left(\frac{x}{y}\right) = \frac{y \cdot 1 - x \cdot dy/dx}{y^2}$

$\frac{d}{dx}(\sin^2 xy) = 2\sin xy \times \cos xy \times$

Q. $x^3 + x^2y + xy^2 + y^3 = 81$

$= 3x^2 + \left[x^2 \cdot \frac{dy}{dx} + y \cdot 2x\right] + \left[x \cdot 2y\frac{dy}{dx} + y^2 \cdot 1\right]$

$+ 3y^2\frac{dy}{dx} = 0$

Ex 5.3
1 to 8

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Imp

Q. if $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$

show that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$

① Chain Rule

②

$$\frac{dy}{dx} = \left[\frac{1}{2\sqrt{1-x^2}} \times -2x\right] + \left[\frac{1}{2\sqrt{1-y^2}} \times -2y\frac{dy}{dx}\right] =$$

$$= a\left[1 - \frac{dy}{dx}\right] + (x-y)[1]$$

$$\frac{dy}{dx} = \left[\frac{-2x}{2\sqrt{1-x^2}} + \frac{-2y}{2\sqrt{1-y^2}}\right.$$

Substitution method

Put $x = \sin\theta \quad y = \sin\phi$

$$\frac{\cos\theta + \cos\phi}{\sin\theta - \sin\phi} = a$$

$$\frac{2\cos\left(\frac{\theta+\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right)}{2\cos\left(\frac{\theta+\phi}{2}\right)\sin\left(\frac{\theta-\phi}{2}\right)} = a$$

$$\theta = \phi = 2\cot^{-1}a$$

$$\sin^{-1}x - \sin^{-1}y = 2\cot^{-1}x$$

$$\frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = \frac{-2}{1+x^2}$$

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Ques

if $x\sqrt{1+y} + y\sqrt{1+x} = 0$ $\quad \frac{dy}{dx} = -\frac{1}{(1+x)^2}$

$x\sqrt{1+y} = -y\sqrt{1+x}$

Path for Success

$x^2(1+y) = y^2(1+x)$ → $x^2 + x^2y = y^2 + y^2x$

$(x^2 - y^2) + xy(x-y) = 0$ ← $x^2 - y^2 + x^2y - y^2x = 0$

$(x-y)[x+y+xy] = 0$

$x - y \neq 0$

$x \neq y$

$x + y + xy = 0$

$y(1+x) = -x$

$y = -\frac{x}{1+x}\left(\frac{u}{v}\right) = \frac{-1(1+x) + x(1)}{(1+x)^2} = \frac{-1-x+x}{(1+x)^2}$

$= \frac{-1}{(1+x)^2}$

Diff.

$\frac{dy}{dx} = -\frac{1}{(1+x)^2}$

Ques: $x\cos(a+y) = \cos y$ $\quad \frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$

$x = \frac{\cos y}{\cos(a+y)} \cdot \left(\frac{u}{v}\right)$

$\frac{dx}{dy} = \frac{\cos(a+y)\sin y + \cos y \sin(a+y)}{\cos^2(a+y)}$

$\frac{dx}{dy} = \frac{\sin[a+y-y]}{\cos^2(a+y)}$

$\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$

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Ques: $x \sin(a+y) + \sin a \cos(a+y) = 0$

$$\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$$

$$x \sin(a+y) = -\sin a \cos(a+y)$$

$$x = -\frac{\sin a \cos(a+y)}{\sin(a+y)} \quad \left(\frac{u}{v}\right)$$

$$\frac{dx}{dy} = -\frac{\sin(a+y) \cdot \cos a \cos(a+y)}{}$$

$$x = -\sin a \cot(a+y)$$

$$\frac{dx}{dy} = -\sin a \cdot x - \operatorname{cosec}^2(a+y)$$

$$\frac{dx}{dy} = -\frac{\sin a}{\sin^2(a+y)}$$

$$\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$$

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Method (Suitable)

* Diff. of I-f function. by Substitution method :

Expression	Sub.
$\sqrt{1-x^2}$	$x = \sin\theta$, $\cos\theta$
$\sqrt{1+x^2}$	$x = \tan\theta$, $\cot\theta$
$\sqrt{x^2-1}$	$x = \sec\theta$, $\operatorname{cosec}\theta$
$\sqrt{\frac{1 \pm x}{1 \mp x}}$	$x = \cos 2\theta$
$\sqrt{\frac{1 \pm x^2}{1 \mp x^2}}$	$x^2 = \cos 2\theta$

Eg. F $\sin^{-1}(\sin x) = x$

$\sin^{-1}\left(\frac{1}{x}\right) = \operatorname{cosec}^{-1} x$.

$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$

P.No. 42 (i) $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left[\frac{x \mp y}{1 \mp xy}\right]$

Eg.

/

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$1+\cos 2\theta = 2\cos^2\theta$

$$y = \tan^{-1}\left[\frac{3x - x^3}{1-3x^2}\right]$$

Put $x = \tan\theta$

$$\theta = \tan^{-1}x$$

$$y = \tan^{-1}\left[\frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}\right]$$

$$y = \tan^{-1}[\tan 3\theta]$$

$$y = 3\theta = 3\times\tan^{-1}x$$

$$\frac{dy}{dx} = \frac{3}{1+x^2}$$

Q. $y = \tan^{-1}\left[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right]$ $\quad -1 < x < 1$

$\left[x^2 = \frac{\cos 2\theta}{\sin 2\theta}\right]$

Put $x^2 = \cos 2\theta \Rightarrow \theta = \frac{1}{2}\cos^{-1}x^2$

$$y = \tan^{-1}\left[\frac{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}\right]$$

$$\Rightarrow y = \tan^{-1}\left[\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}\right]$$

[$\because \cos 2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$]

$$\Rightarrow \tan^{-1}\left[\frac{1+\tan\theta}{1-\tan\theta}\right]$$

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$= \tan^{-1}\left[\tan\left(\frac{\pi}{4}+\theta\right)\right]$

$-1 < x < 1$
$0 < x^2 < 1$
$0 < \cos 2\theta < 1$
$0 < 2\theta < \frac{\pi}{2}$
$0 < \theta < \frac{\pi}{4}$.

$y = \frac{\pi}{4} + \theta$

$y = \frac{\pi}{4} + \frac{1}{2}\cos^{-1}x^2$

$\frac{dy}{dx} = 0 + \frac{1}{2} \times \frac{-1}{\sqrt{1-x^4}}(2x)$

$\frac{dy}{dx} = \frac{-x}{\sqrt{1-x^4}}$ Ans.

Q. $y = \cos^{-1}\left[\frac{3x + 4\sqrt{1-x^2}}{5}\right]$

$y = \cos^{-1}\left[\frac{3}{5}x + \frac{4}{5}\sqrt{1-x^2}\right]$

Put $x = \sin\theta$ $\quad \frac{3}{5} = \sin\phi$ ($\cos\phi$)

$y = \cos^{-1}[\sin\theta\sin\phi + \cos\theta\cos\phi]$

$y = \cos^{-1}(\cos(\phi-\theta))$

Ques: $y = \sin^{-1}\left[\frac{6x - 4\sqrt{1-4x^2}}{5}\right]$

$y = \sin^{-1}\left[\frac{3}{5}(2x) - \frac{4}{5}\sqrt{1-(2x)^2}\right]$

Put $2x = \sin\theta$ $\quad \frac{3}{5} = \cos\phi$

$= \sin^{-1}(\sin(\theta-\phi))$

$y = \theta - \phi$

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$$y = \sin^{-1} 2x - \cos^{-1} \frac{3}{5}$$

(5.5 and 5.6)

Diff. of Logarithmic function:

$\log(a+b) \neq \log a + \log b$

$$y = [f(x)]^{g(x)}$$

Q. $y = x^x$

P.D

$\log y = \log x^x$

$\log y = x \times \log x$

$\frac{dy}{dx}\frac{1}{y} = x\frac{1}{x} + \log x$

$y = e^{\log x^x}$

$y = e^{x \log x}$.

$y = x x^{x-1} + x^x \log x$

Que:1) $y = x^{\sin x} + (\sin x)^{\cos x}$

find $\frac{dy}{dx}$.

$= u + v$

$u = x^{\sin x}$ $v = \sin x^{\cos x}$

(2) $x^y + y^x = 1$

(3) $x^y = y^x$

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Q. $y = x^{\sin x} + (\sin x)^{\cos x}$

$y = u + v$

Diff^{u}.

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \quad \text{(1)}$$

$u = x^{\sin x}$

$\log u = \sin x \log x$

$$\frac{1}{u}\frac{du}{dx} = \left[\frac{\sin x}{x} + \cos x \cdot \log x\right]$$

$$\frac{du}{dx} = x^{\sin x}\left[\quad \text{''} \quad\right]$$

$v = (\sin x)^{\cos x}$

$\log v = \cos x \log \sin x.$

$$\frac{1}{v}\frac{dv}{dt} = \frac{\cos x}{\sin x} - \sin x \log \sin x$$

$$\frac{dv}{dt} = (\sin x)^{\cos x}\left[\quad \text{''} \quad\right]$$

Q. if $y^x = e^{y-x}$. $\quad \frac{dy}{dx} = \frac{(1+\log y)^2}{\log y}$

$= x \log y = y - x \log e$

$x \log y = y - x$

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$x(\log y + 1) = y$

$x = \frac{y}{1+\log y}$

$\frac{dx}{dy} = $ ——

$\frac{dx}{dy} = \frac{y\left(\frac{1}{y}\right) - 1\cdot(1+\log y)}{(1+\log y)^2}$

Ex : 5.7

Diff. of Parametric form :

$x = a\cos\theta \qquad y = a\sin\theta$

Do not use elimination method of parameter.

$\frac{dx}{d\theta} = -a\sin\theta$; $\frac{dy}{d\theta} = a\cos\theta$ — ②

— ①

Now eqn ② / ①

$\frac{dy/d\theta}{dx/d\theta} = \frac{a\cos\theta}{-a\sin\theta} = -\cot\theta$

$\frac{dy}{dx} = -\cot\theta$

Q. find $\frac{d^2y}{dx^2}$ $\qquad \frac{d^2y}{dx^2} = -\left(-\operatorname{cosec}^2\theta\right) \times \frac{d\theta}{dx}$

from eq 1

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$$\frac{d^2y}{dx^2} = \operatorname{cosec}^2\theta \times -\frac{1}{a\sin\theta}$$

$$\frac{d^2y}{dx^2} = -\frac{1}{a}\operatorname{cosec}^3\theta.$$

Q.5.6
Q.8
Av. Smk

$x = a\left(\cos t + \log \tan\frac{t}{2}\right)$ $\quad y = a\sin t.$

Find $\left(\frac{d^2y}{dx^2}\right)_{\frac{\pi}{4}}$

$$\frac{dx}{dt} = a\left(-\sin t + \frac{1}{\tan t/2} \times \sec^2\frac{t}{2}\times\frac{1}{2}\right) \quad -(1)$$

$$\frac{dy}{dt} = a\cos t. \quad -(2)$$

$$\frac{dy/dt}{dx/dt} = \frac{a\left(-\sin t + \frac{1}{\tan t/2}\times\sec^2 t/2\times\frac{1}{2}\right)}{a\cos t}$$

$$\frac{dy}{dx} = \frac{-\sin t + \frac{\sec^2 t/2}{\tan t/2}}{\cos t}.$$

$$\frac{d^2y}{dx^2} =$$

$$\frac{dx}{dt} = a\left(-\sin t + \frac{1}{a\sin\frac{t}{2}\cos\frac{t}{2}}\right)$$

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$= a\left(-\sin t + \frac{1}{\sin t}\right)$

$\frac{dx}{dt} = a\left(\frac{-\sin^2 t + 1}{\sin t}\right) = \frac{a\cos^2 t}{\sin t}$ — (1)

$\frac{dy}{dt} = a\cos t$ — (2)

(2)/(1) $\frac{dy}{dx} = \frac{a\cos t \times \sin t}{a\cos^2 t}$

$\frac{dy}{dx} = t\tan t$

$\frac{d^2y}{dx^2} = \sec^2 t \times \left(\frac{dt}{dx}\right)$

$= \sec^2 t \times \frac{\sin t}{a\cos^2 t}$.

Q. 5.6
$a = 11$ If $x = a\sin 2t\,(1+\cos 2t)$
$y = b\cos 2t\,(1-\cos 2t)$

then show that

$\left(\frac{dy}{dx}\right) = \frac{b}{a}$ at $t = \frac{\pi}{4}$

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$y = b\cos\omega t(1-\cos\omega t)$

$\frac{dy}{dx} = y[$

$y = b\cos\omega t - b\cos^2\omega t$

$\frac{dy}{dt} = -b\sin\omega t + (2b\cos\omega t \times b\sin\omega t)$

$\frac{dy}{dt} = b\sin\omega t(2b\cos\omega t - \omega)$

$x = a\sin\omega t(1+\cos\omega t)$ $\quad \frac{a}{2}(\sin 4t)$

$= a\sin\omega t + a\sin\omega t.\cos\omega t$

$\frac{dx}{dt} = 2a\cos\omega t + \frac{a}{2}[4\cos 4t]\ a(\cos^2\omega t + \sin^2\omega t)$

$= 2a\cos\omega t + a\cos^2\omega t - a\sin^2\omega t$

$= 2a\cos\omega t + a(1-\sin^2 2t) - a\sin^2\omega t$

$=$

$1-\sin^2 t = \cos^2 t$

But

$\therefore$ L.

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1. Matrix
2. Relation & function
3. LTE
4. Determinant

3

Determinant

* **Determinant :-**

To every square matrix $A = [a_{ij}]$ of order n, we can associate a no. (real or complex) called determinant of the square matrix A, where a_{ij} = $(i, j)^{th}$ element of A.

① Determinant of order one

$A = [a]$

② Determinat of matrix order two

Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

then $\det(A) = |A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$

$= (a_{11}a_{22} - a_{21}a_{12})$ Ans

example:

Evaluate $\begin{vmatrix} 2 & 4 \\ -1 & 2 \end{vmatrix} = 4 + 4 = 8.$

③ Determinant of order three.

Ex:- $\begin{vmatrix} -1 & -2 & 4 \\ -1 & 3 & 0 \\ 4 & 1 & 0 \end{vmatrix}$.

$1(0-0) - 2(0-0) + 4(-1-12)$

$= -52$ Ans

Properties:-

① The value of det. remains unchanged if its row and column are interchanged.

Ex. $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix} = -28 = \begin{vmatrix} 2 & 6 & 1 \\ -3 & 0 & 5 \\ 5 & 4 & -7 \end{vmatrix}$

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(2) If any two rows (or columns) of a det. are interchanged then sign of determinant changes.

Ex: $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix} = -28$

if we interchange two rows or columns.

$\begin{vmatrix} 2 & -3 & 5 \\ 1 & 5 & -7 \\ 6 & 0 & 4 \end{vmatrix} = 28$

(3) If any two rows (or columns) of a det. are identical (all corresponding elements are same) then value of det. zero.

Ex: $= \begin{vmatrix} 3 & 2 & 3 \\ 2 & 2 & 3 \\ 3 & 2 & 3 \end{vmatrix} = 0.$

R_1 and R_3 are same.

(4) If each element of a row (or a column) of a det. is multiply by a constant K, then its value get multiply by K.

of det

Ex: $\begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix} = \begin{vmatrix} 6(17) & 6(3) & 6(6) \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix} = 6 \begin{vmatrix} 17 & 36 \\ 1 & 34 \\ 17 & 36 \end{vmatrix} =$ (det)

(5) If some or all elements of a row or column of a det. are expressed as sum of two (or more) terms then the det. can be expressed as sum of two (or more) det.

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For Ex:

$$\begin{vmatrix} a_1+d_1 & a_2+d_2 & a_3+d_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} + \begin{vmatrix} d_1 & d_2 & d_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

(6) If to each elements of any row or column of a det., the equimultiples of corresponding element of other row (or column) are added, then value of det. remains the same

the value of det. remains same if we apply the operations

$$R_i^\circ \to R_i^\circ + kR_j^\circ \quad , \quad C_i \to C_i^\circ + kC_j^\circ.$$

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13/11/17

Chapter - 10

Vector

17.1.

7 mark vectors [1+1+1+2+2]

3D 10 mark 4+6

Ex- 10.1 and 10.2

Distance - S - km/hr

Displacement - V - 10 km/hr to ward any Clair

Speed - S -

velocity. - V -

Work - S -

force. - V -

volume - S -

Density - S -

time Period - S -

- A quantity that has magnitude as well as dirn is called vector

unit vector $\hat{a} = \frac{\vec{a}}{|\vec{a}|}$

- if $P(x, y, z)$ is a point then position vector $\vec{OP} = x\hat{i} + y\hat{j} + zk$.

and magnitude $|\vec{OP}| = \sqrt{x^2+y^2+z^2}$

- Scalar components are direction ratio's.

(3 sides of Δ sum = 0)

* Types of vectors :-

1) null vector : $|\vec{AA}| = 0$

2) ☆ Unit vector : unit vector in the dirn of the $\vec{A}$

$$\hat{a} = \frac{\vec{a}}{|a|}$$

eg : find the unit vector in the dirn of $i^\circ - 2j + 5k$.

$$\vec{a} = \frac{i^\circ - 2j + 5}{\sqrt{1^2+2^2+5^2}} \quad \text{Ans}$$

eg :

Eg: find ~~the~~ a vect. along the vector $\vec{a}+\vec{b}$ where $\vec{a} = i - 2j + 6k$

$\vec{b} = 2i + 3j + 5k$

$$\hat{a} = \frac{\vec{a}+\vec{b}}{|\vec{a}+\vec{b}|} = \frac{3i + j + 11k}{\sqrt{9+1+121}}$$

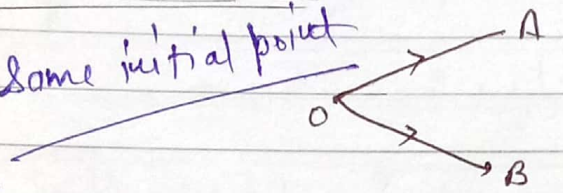
(1) Addition

(2)

(3)

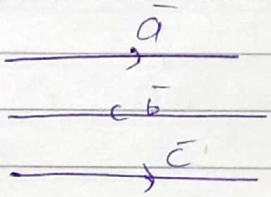
* Coinitial vectors:

Same initial point



Imp.

**** Collinear vectors (Parallel vectors):



(1) $\vec{a} = \lambda\vec{b}$ — a or b are collinear

(2) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\vec{a} = 2i - j + 6$

$\vec{b} = 4i - 2j + 12k$

$\vec{b} = 2\vec{a}$

or $\frac{2}{4} = \frac{-1}{-2} = \frac{6}{12} = \frac{1}{2}$

* A(-1, 2, 3) B(0, 1, 5) C(-2, 4, 6)

$\vec{AB} = B - A$

$\vec{BC} = C - B$

$\vec{AB} = \lambda\vec{BC}$

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(3)

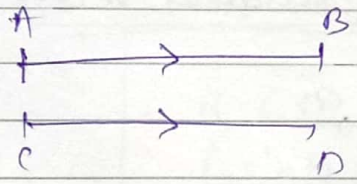
(1) (i) Two vectors $\vec{A}$ and $\vec{B}$ are collinear if $\vec{a} = \lambda \vec{b}$ or

(ii) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

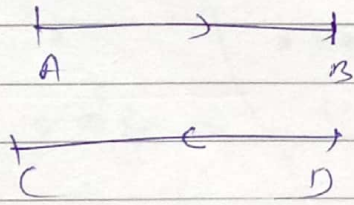
(2) 3 Point with Position vector if

$$|\vec{AB}| + |\vec{BC}| = |\vec{AC}|$$

* Equal Ved. :-

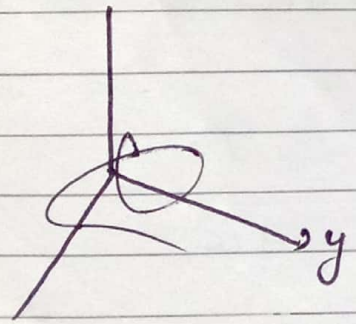


* Negtive :-

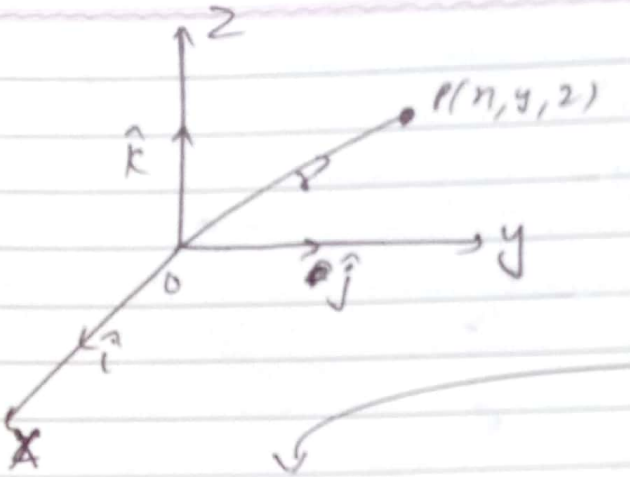


Excercise - 10.2

Position vector, Component of a Vectors, magnitude



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Position vector

if P is any point then P.V

$\vec{r} = \vec{OP} = x\hat{i} + y\hat{j} + z\hat{k}$

and magnitude $r = |\vec{OP}| = \sqrt{x^2+y^2+z^2}$

Vector componant = $x\hat{i}, y\hat{j}, z\hat{k}$

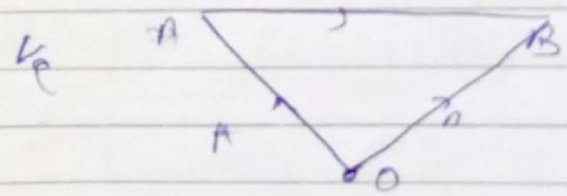
Scalar componant = x, y, z

Eg: write the V.C/S.C of a vector $\vec{r} = 2\hat{i} - 3\hat{j} + 6\hat{k}$

V.C = $2\hat{i}, -3\hat{j}, 6\hat{k}$

S.C = $2, -3, 6$

2) Vector joining two points:

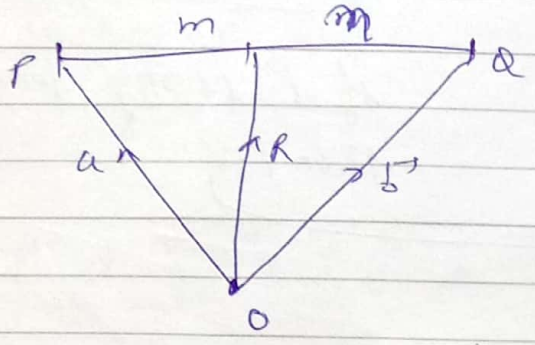


$\vec{AB} = \vec{OB} - \vec{OA}$ = P.V of B − P.V of A

$\vec{AB} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$

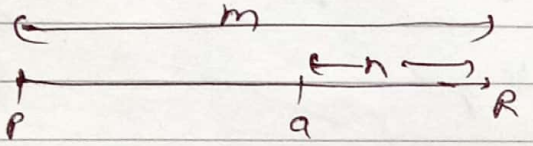
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(3) Division.



$$\vec{OR} = \frac{m\vec{b} + n\vec{a}}{m+n}$$ internally

(4) External Division.



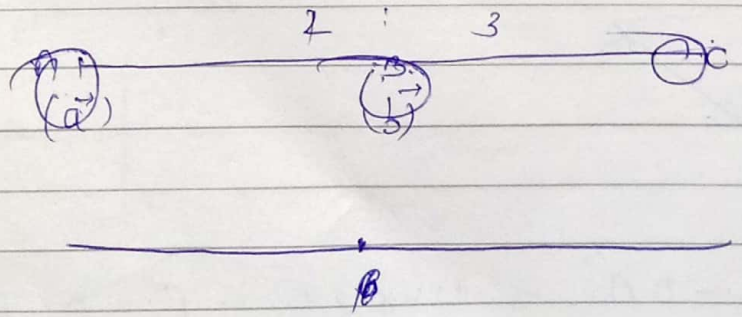
externally

$$\vec{OR} = \frac{m\vec{b} - n\vec{a}}{m-n}.$$

Ques: if $\vec{a}$ and $\vec{b}$ are the P.v of $\vec{A}$ and $\vec{B}$ respectively find the P.v of a $\vec{C}$ in $\vec{BA}$ Produce such that

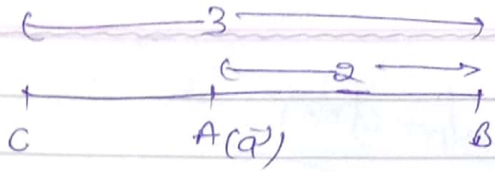
$$\vec{BC} = 1.5\,\vec{BA}$$

$\frac{15}{10}$ $\frac{3}{2}$



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Ans:-



$$\frac{\vec{BC}}{\vec{BA}} = \frac{3}{2}$$

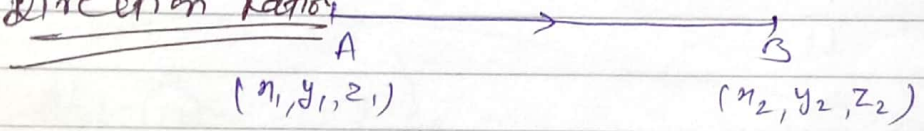
$$\frac{\vec{OC} - \vec{OB}}{\vec{OA} - \vec{OB}} = \frac{3}{2}$$

$$\frac{\vec{OC} - \vec{b}}{\vec{a} - \vec{b}} = \frac{3}{2}$$

$$\vec{OC} =$$

* Direction Ratio and Direction Cosine of a Vector:-

Direction Ratio:



$$\vec{AB} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

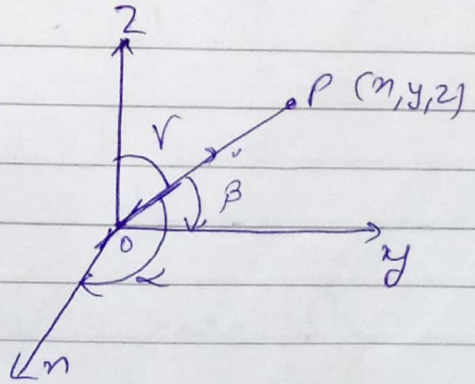
D.R ⇒ $a = (x_2 - x_1$
$b = y_2 - y_1$
$c = z_2 - z_1$

Eg ⇒ $\vec{r} = 2\hat{i} - 3\hat{j} + 7\hat{k}$

D.R ⇒ $a = 2$, $b = -3$, $c = 7$

Direction Cosine:-

$l = \cos\alpha$
$m = \cos\beta$
$n = \cos\gamma$
$l^2 + m^2 + n^2 = 1$



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$M \cdot \omega = 10 \cdot 2$ $\cos\frac{\pi}{3}$.

***

** if a, b, c are given direction cosine then

$$l = \pm \frac{a}{\sqrt{a^2+b^2+c^2}} \qquad m = \pm \frac{b}{\sqrt{a^2+b^4+c^2}} \qquad n = \pm \frac{c}{\sqrt{a^2+b^2+c^2}}$$

↑ ~~To Both acute~~

acute = +ve

obtuse = -ve

Q.1 Show that $\hat{i}+\hat{j}+\hat{k}$ is equally inclined to the axis.

$$a = b = c = 1$$

$$l = \frac{a}{\sqrt{a^2+b^2+c^2}} = \frac{1}{\sqrt{3}}$$

$$l = m = n.$$

$$\cos\alpha = \cos\beta = \cos\gamma = \phi$$

$$\alpha = \beta = \gamma$$

$$\vec{r} = \frac{1}{\sqrt{3}}(\pm\hat{i} \pm \hat{j} \pm \hat{k})$$

$$\vec{r} = \frac{6\sqrt{3}}{\sqrt{3}}(\pm\hat{i} \pm \hat{j} \pm \hat{k}).$$

$$l = \cos\alpha = \cos\frac{\pi}{3} = \frac{1}{2}$$

$$m = \cos\beta = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

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10.2 Q. 6, 9, 10, 11, 14, 15, 17 Imp.

$n = \cos\gamma$

$l^2 + m^2 + n^2 = 1$

$\frac{1}{4} + \frac{1}{2} + \cos^2\gamma = 1$

$\cos^2\gamma = \frac{1}{4}$

$\cos\gamma = \frac{1}{2}$

$\gamma = \frac{\pi}{3}$

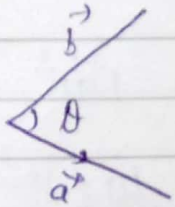
$\vec{r} = l\hat{i} + m\hat{j} + n\hat{k}$

$= \left(\frac{1}{2}\hat{i} + \frac{1}{\sqrt{2}}\hat{j} \pm \frac{1}{2}\hat{k}\right)$ Ans.

Hint: (17) - find AB and BC + CA and $|AB|$, $|BC|$, $|CA|^2$.

Exercise - 10.3

⊛ DOT (Scalar) Product :



1) $\vec{a} \cdot \vec{b} = |\vec{a}|\,|\vec{b}| \cos\theta$

$= ab\cos\theta$

2) $\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|\,|\vec{b}|}$

i) $\theta = 0 \Rightarrow \vec{a}$ is $\parallel$ to $\vec{b} \Rightarrow \vec{a} \cdot \vec{b} = |\vec{a}|\,|\vec{b}|$

ii) $\theta = 90^\circ \Rightarrow \vec{a}$ is $\perp^r$ to $\vec{b} = \vec{a} \cdot \vec{b} = 0$

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(3) Projection of $\vec{a}$ on $\vec{b}$ = $\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$
along

(4) $\vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$

$\vec{b} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$

$\hat{i}\cdot\hat{i} = j\cdot j = k\cdot k = 0$

$\hat{i}\cdot j = j\cdot k = k\cdot i = 0$

$\vec{a}\cdot\vec{b} \Rightarrow a_1a_2 + b_1b_2 + c_1c_2$

$a \parallel b = a_1/a_2 = b_1/b_2 = c_1/c_2$

$a \perp b \Rightarrow a_1a_2 + b_1b_2 + c_1c_2 = 0$

(5)

(i) $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$ (Comm. Rule)

(ii) $\vec{a}\cdot(\vec{b}+\vec{c}) = \vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{c}$

(3) $|\vec{a}|^2 = \vec{a}\cdot\vec{a}$

$|\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}\cdot\vec{b}|$

$|\vec{a}-\vec{b}|^2 =$

$(\vec{a}+\vec{b})(\vec{a}-\vec{b}) = |\vec{a}|^2 - |\vec{b}|^2$

$|\vec{a}+\vec{b}+\vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(a\cdot b + b\cdot c + c\cdot a)$

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Q.4 $\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$ = $\vec{a} = i + 3j + 7k$ $\quad b = 7i - j + 8k$

$= \frac{7 - 3 + 56}{\sqrt{49+1+64}} = \frac{60}{\sqrt{114}}$ Ans

(5) $\vec{a} = \frac{1}{7}(2i + 3j + 6k)$ $\quad \vec{b} = \frac{1}{7}(3i - 6j + 2k)$

$\vec{c} = \frac{1}{7}(6i + 2j - 3k)$

$|\vec{a}| = \frac{1}{7}\sqrt{4+9+36} = \frac{1}{7} \times \sqrt{49} = \frac{1}{7} \times 7 = 1$

$|\vec{b}| = \frac{1}{7}\sqrt{9+36+4} = \frac{1}{7} \times \sqrt{49} = \frac{1}{7} \times 7 = 1$

$|\vec{c}| = \frac{1}{7}\sqrt{36+4+9} = \frac{1}{7} \times \sqrt{49} = \frac{1}{7} \times 7 = 1$

Hence $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$

$\vec{a} \cdot \vec{b} = 0$

6. $|\vec{a}| = 8|\vec{b}|$, $(\vec{a}+\vec{b}) \cdot (\vec{a}-\vec{b}) = 8$

$|\vec{a}|^2 - |\vec{b}|^2 = 8$

$64|\vec{b}|^2 - |\vec{b}|^2 = 8$

$63|\vec{b}|^2 = 8$

$|\vec{b}| = \sqrt{8/63}$

$|\vec{a}| = 8\sqrt{8/63}$

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Ex = 28, 29

Q.17 $(\vec{r}-\vec{a})(\vec{r}+\vec{a}) = 12$

$|\vec{r}| = ?$

$|\vec{a}| = 1$

$|\vec{r}|^2 - |\vec{a}|^2 = 12$

$= 12+1$

$r = \sqrt{13}$

Q.18

$|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$

$|\vec{a}|^2 = \vec{a}\cdot\vec{a}$

$\vec{a}+\vec{b}+\vec{c} = \vec{0}$

$|\vec{a}+\vec{b}+\vec{c}|^2 = |\vec{0}|^2$

$(\vec{a}+\vec{b}+\vec{c})(\vec{a}+\vec{b}+\vec{c}) = 0$

$= |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = 6$

$3 + 2(\quad) = 0$

$(\quad) = -3/2$

Ex: 28 $|\vec{a}| = 3 \quad |\vec{b}| = 4 \quad |\vec{c}| = 5$

$\vec{a}\cdot(\vec{b}+\vec{c}) = 0$ $\quad \vec{b}\cdot(\vec{c}+\vec{a}) = 0$ $\quad \vec{c}(\vec{a}+\vec{b}) = 0$

$a\cdot b + a\cdot c = 0$ - (1) $\quad b\cdot c + b\cdot a = 0$ - (2) $\quad$ - (3)

$|\vec{a}+\vec{b}+\vec{c}| = 0 \Rightarrow |\vec{a}+\vec{b}+\vec{c}|^2 = 0$

$= (\vec{a}+\vec{b}+\vec{c})(\vec{a}+\vec{b}+\vec{c}) = 0$

$|a+b+c|^2 = |\vec{a}|^2 + |b|^2 + |\vec{c}|^2 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a})$

$|a+b+c| = \sqrt{(8)^2 + (4)^2 + (5)^2} \neq 0$

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Ex 29

$|\vec{a}| = 3 \quad |\vec{b}| = 4 \quad |\vec{c}| = 20$

$\vec{a}+\vec{b}+\vec{c} = 0 \qquad \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a} = ?$

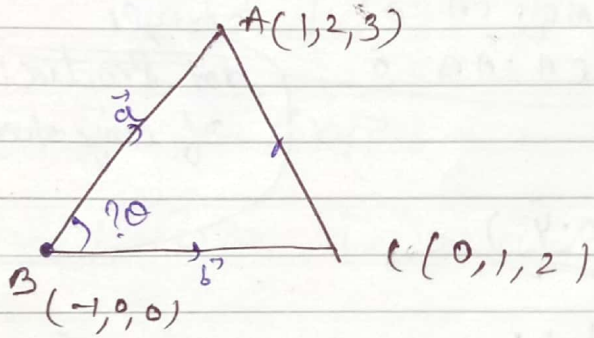
$|\vec{a}+\vec{b}+\vec{c}|^2 = |0|^2$

$|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = 0$

$= (3)^2 + (4)^2 + (20)^2 + 2(\quad) = 0$

$(\quad) = 2$

Q.15



$\vec{A} = \hat{i} + 2\hat{j} + 3\hat{k} \qquad \vec{B} = -\hat{i} + 0\hat{j} + 0\hat{k} \qquad \vec{C} = 0\hat{i} + \hat{j} + 2\hat{k}$

$\angle ABC$ = angle b/w $\vec{BA}$, $\vec{BC}$

$\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}|\,|\vec{b}|}$

$\vec{a} = \vec{BA} = 2\hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{b} \Rightarrow \vec{BC} = \hat{i} + \hat{j} + 2\hat{k}$

$\cos\theta = \dfrac{2+2+6}{\sqrt{4+4+9}\,\sqrt{1+1+4}} \Rightarrow \cos\theta = \dfrac{10}{\sqrt{17}\,\sqrt{6}}$

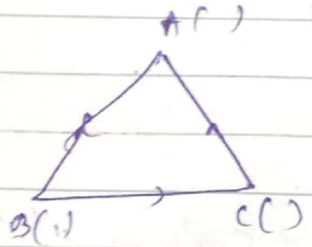
$\cos\theta = \dfrac{10}{\sqrt{102}}$

$\theta = \cos^{-1}\left(\dfrac{10}{\sqrt{102}}\right)$

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(16.) show that 3 Points are collinear $\frac{a_1}{a_2} + \frac{b_1}{b_2} = \frac{c_1}{c_2}$

(11.)



$\vec{AB} =$

$\vec{BC} =$

$\vec{CA} =$

$u = \frac{1}{2}b$

$\vec{AB} + \vec{BC} + \vec{CA} = 0$

ABC

shows 3 vertices are of the triangle

square the three vector and Proff with Pythagoreus theorem

$\vec{A} \cdot \vec{B}$

$\vec{AB} \cdot \vec{BC} = 0$

$\vec{BC} \cdot \vec{CA} = 0$

$\vec{CA} \cdot \vec{AB} = 0$

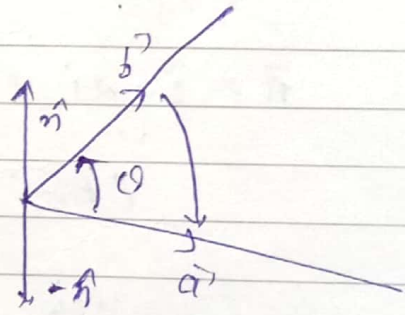
and for Right angle triangle dot Product is zero of any two vector

(10.4)

* Cross (vector Product) :

(1) $(\vec{a} \times \vec{b}) = |\vec{a}||\vec{b}| \sin\theta \cdot \hat{n}$

(2) $\sin\theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|}$ $\quad |\hat{n}| = 1$

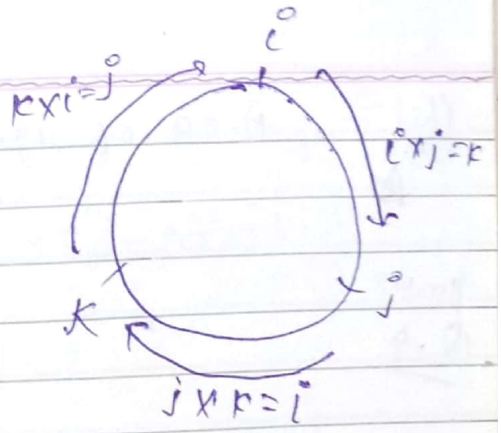


(3) unit vector $\hat{n} = \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|} = \frac{\vec{c}}{|\vec{c}|}$

(4) $\theta = 0 \Rightarrow a \parallel b \Rightarrow |\vec{a} \times \vec{b}| = 0$

$\theta = 90^\circ \Rightarrow a \perp b \Rightarrow |\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}|$

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(5) $\vec{a} = a_1 i + b_1 j + c_1 k$

$\vec{b} = a_2 i + b_2 j + c_2 k$

$i \times i = j \times j = k \times k = 0$

$\vec{a} \times \vec{a} = \vec{b} \times \vec{b} = \vec{c} \times \vec{c} = 0$

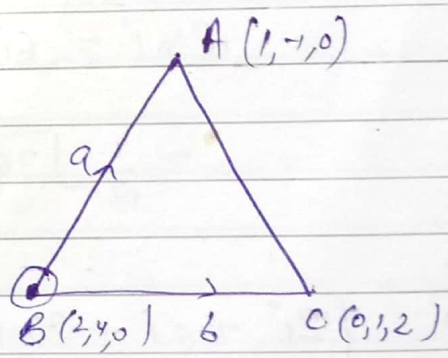
$$\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

(6) (1) $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$

(2) $\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$

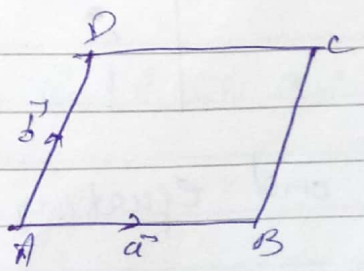
(3) Area of Δ:

$$\Delta = \frac{1}{2} |\vec{a} \times \vec{b}|$$



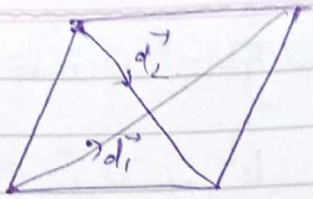
(4) Area of Parallogram:

$$= |\vec{a} \times \vec{b}|$$



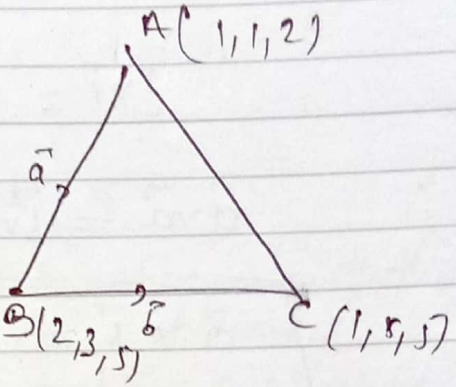
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(b) Area of ||gm $= \frac{1}{2}|\vec{d_1} \times \vec{d_2}|$



(10.41)

Q.9



$\vec{a} = -\hat{i} - 2\hat{j} - 3\hat{k}$

$\vec{b} = -\hat{i} + 2\hat{j} + 0\hat{k}$.

Area of $\Delta = \frac{1}{2}|\vec{a} \times \vec{b}|$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -2 & -3 \\ -1 & 2 & 0 \end{vmatrix} = 6\hat{i} + 3\hat{j} - 4\hat{k}$$

$\vec{a} \times \vec{b} = 6\hat{i} + 3\hat{j} - 4\hat{k}$.

$$= \frac{1}{2}\sqrt{36+9+16} = \frac{1}{2}\sqrt{61} = \frac{\sqrt{61}}{2}$$

Q.5 $(2\hat{i} + 6\hat{j} + 27\hat{k}) \times (\hat{i} + \lambda\hat{j} + \mu\hat{k}) = \vec{0}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 6 & 27 \\ 1 & \lambda & \mu \end{vmatrix} = 0\hat{i} + 0\hat{j} + 0\hat{k}$$

and equating the value of $\hat{i}, \hat{j}, \hat{k}$ and find μ and λ.

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Imp

* Scalar triple Product :- (STP) or

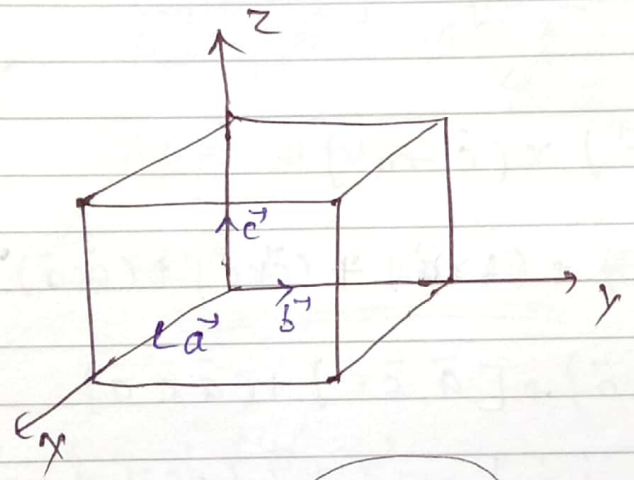
$$[\vec{a}\ \vec{b}\ \vec{c}] = \vec{a}\cdot(\vec{b}\times\vec{c}) = (\vec{a}\times\vec{b})\cdot\vec{c}$$

*

① Volume of Parallo Pipe = $[\vec{a}\ \vec{b}\ \vec{c}]$

$$\Rightarrow \vec{a}\cdot(\vec{b}\times\vec{c}) = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$[(\vec{a}\times\vec{b})\cdot\vec{c}\,] = \vec{c}\cdot(\vec{a}\times\vec{b}) = \begin{vmatrix} a_3 & b_3 & c_3 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$



* *

② Three vector $\vec{a}\ \vec{b}\ \vec{c}$ are Coplanar =

$$[\vec{a}\ \vec{b}\ \vec{c}] = 0$$

③ four points with Position vector A, B, C, D are Coplanar = $[\vec{AB}\ \ \vec{AC}\ \ \vec{AD}] = 0$

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q) (ii) $[\vec{a}\ \vec{b}\ \vec{c}] = [\vec{b}\ \vec{c}\ \vec{a}] = [\vec{c}\ \vec{a}\ \vec{b}]$

(iii) $[\vec{a}\ \vec{a}\ \vec{b}] = [\vec{a}\ \vec{b}\ \vec{b}] = [\vec{a}\ \vec{c}\ \vec{c}] = 0$

(iii) $[\hat{i}\ \hat{j}\ \hat{k}] = 1 = [\hat{j}\ \hat{k}\ \hat{i}] = [\hat{k}\ \hat{i}\ \hat{j}]$

(iv) $[\hat{i}\ \hat{k}\ \hat{j}] = -1$

Very Important

Q. Show that (1X) (P. 19 x R)

$$[\underbrace{\vec{a}+\vec{b}}_{p} \cdot \underbrace{(\vec{b}+\vec{c}}_{q} \times \underbrace{\vec{c}+\vec{a})}_{r}] = 2[\vec{a}\ \vec{b}\ \vec{c}]$$

Proof:

$$(\vec{a}+\vec{b}) \cdot [(\vec{b}+\vec{c}) \times (\vec{c}+\vec{a})]$$

$$= (\vec{a}+\vec{b}) \cdot [(\vec{b}\times\vec{c}) + (\vec{b}\times\vec{a}) + (\vec{c}\times\vec{c}) + (\vec{c}\times\vec{a})]$$

$$= [\vec{a}\ \vec{b}\ \vec{c}] + [\vec{a}\ \vec{b}\ \vec{a}] + [\vec{a}\ \vec{c}\ \vec{c}] + [\vec{a}\ \vec{c}\ \vec{a}]$$
$$+ [\vec{b}\ \vec{b}\ \vec{c}] + [\vec{b}\ \vec{b}\ \vec{a}] + [\vec{b}\ \vec{c}\ \vec{c}] + [\vec{b}\ \vec{c}\ \vec{a}]$$

$$= [\vec{a}\ \vec{b}\ \vec{c}] + [\vec{b}\ \vec{c}\ \vec{a}]$$

$$= [\vec{a}\ \vec{b}\ \vec{c}] + [\vec{a}\ \vec{b}\ \vec{c}]$$

$$= 2[\vec{a}\ \vec{b}\ \vec{c}]$$

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Misc-Ex-10 6 — 19 All a

Misc Imp 12, 13, 14 highlighted

All Misc Ex-13

(8) $(\hat{i}-\hat{j}+k)$, $(3\hat{i}+\hat{j}+2k)$ $(\hat{i}+\lambda\hat{j}-3k)$

if 3 vectors are coplanar:

$$[\vec{a}\ \vec{b}\ \vec{c}]=0$$

$$= \begin{vmatrix} 1 & -1 & 1 \\ 3 & 1 & 2 \\ 1 & \lambda & -3 \end{vmatrix} = 0 \qquad \lambda = ?$$

Q.6 A(3, 2, 1) B(4, 1, 5) C(4, 2, −2) D(6, 5, −1)

$$\vec{AB} = \vec{OB} - \vec{OA}$$

$$\vec{AC} = \vec{OC} - \vec{OA}$$

$$\vec{AD} = \vec{OD} - \vec{OA}$$

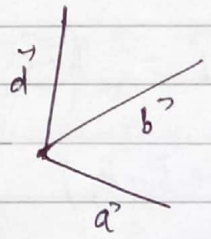
* Misc Ex. 10 *

(13) $\vec{a} = \hat{i}+4\hat{j}+2k$

$\vec{b} = 3\hat{i}-2\hat{j}+7k$.

$\vec{c} = 2\hat{i}-\hat{j}+4k$

$\vec{c}\cdot\vec{d} = 15$



Given: $\vec{d} \perp \vec{a}$ and $\vec{d} \perp \vec{b}$

$\Rightarrow \vec{d} \parallel (\vec{a}\times\vec{b})$

$\Rightarrow \vec{d} = \lambda(\vec{a}\times\vec{b})$

$$\vec{d} = \lambda \begin{vmatrix} \hat{i} & \hat{j} & k \\ 1 & 4 & 2 \\ 3 & -2 & 7 \end{vmatrix}$$

$$\vec{d} = \lambda(32\hat{i}-\hat{j}-14k) \quad -(1)$$

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Note:- $\vec{c}\cdot\vec{d} = 15$

$\Rightarrow (2\hat{i}-\hat{j}+4\hat{k})\cdot\lambda(32\hat{i}-\hat{j}-14\hat{k}) = 15$

$\lambda = \frac{5}{3}$ put in eq (1)

$\vec{d} = \frac{5}{3}(32\hat{i}-\hat{j}-14\hat{k})$

$64\lambda + \lambda - 56\lambda = 15$

$-7\lambda = 11$

$\lambda = -\frac{5}{3}$

Que: (13) let $\vec{a} = \hat{i}+\hat{j}+\hat{k}$

$\vec{b} = 2\hat{i}+4\hat{j}-5\hat{k}$

$\vec{c} = \lambda\hat{i}+2\hat{j}-3\hat{k}$

$$\vec{a}\cdot\frac{(\vec{b}+\vec{c})}{|\vec{b}+\vec{c}|} = 1$$

$$(\hat{i}+\hat{j}+\hat{k})\cdot\frac{(2+\lambda)\hat{i}+6\hat{j}-8\hat{k}}{\sqrt{(2+\lambda)^2+36+64}}$$

$$(\hat{i}+\hat{j}+\hat{k})\cdot\frac{(2+\lambda)\hat{i}+6\hat{j}-8\hat{k}}{\sqrt{(2+\lambda)^2+100}}$$

14 $|\vec{a}| = |\vec{b}| = |\vec{c}|$

$\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{c} = \vec{c}\cdot\vec{a} = 0$

$(\vec{a}+\vec{b}+\vec{c})$ $\vec{a}$, $\vec{b}$ $\vec{c}$

$$\cos\theta = \frac{(\vec{a}+\vec{b}+\vec{c})\cdot\vec{a}}{|\vec{a}+\vec{b}+\vec{c}|\,|\vec{a}|}$$

$$= \frac{|a|^2+\vec{b}\cdot\vec{a}+\vec{c}\cdot\vec{a}}{|\vec{a}+\vec{b}+\vec{c}|\,|\vec{a}|} = \frac{|a|^2}{|a+b+c|\,|a|^2}$$

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$\cos\alpha = \frac{|\vec{a}|}{|\vec{a}+\vec{b}+\vec{c}|}$

$\cos\beta = \frac{|\vec{b}|}{|\vec{a}+\vec{b}+\vec{c}|}$

$\cos\gamma = \frac{|\vec{c}|}{|\vec{a}+\vec{b}+\vec{c}|}$

$\alpha = \beta = \gamma$

$\cos\alpha = \cos\beta = \cos\gamma$

$l = m = n$

$l^2 + m^2 + n^2 = 1$

$3l^2 = 1$

$l = \frac{1}{\sqrt{3}}$

$\alpha = \cos^{-1}\frac{1}{\sqrt{3}} = \beta = \gamma$

Q.1 $|\vec{a} \times \hat{i}|^2 + |\vec{a} \times \hat{j}|^2 + |\vec{a} \times \hat{k}|^2 = 2|\vec{a}|^2$

$|\vec{a}|^2|\hat{i}|^2 + |\vec{a}|^2|\hat{j}|^2 + |\vec{a}|^2|\hat{k}|^2 =$

Q.2 If $\vec{a} = \hat{i}+\hat{j}+\hat{k}$ and $\vec{b} = \hat{j}-\hat{k}$ find $\vec{c}$ such that $\vec{a}\times\vec{c} = \vec{b}$ and $\vec{a}\cdot\vec{c} = 3$

Let $\vec{c} = x\hat{i}+y\hat{j}+z\hat{k}$.

$(\vec{a}\times\vec{c}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ x & y & z \end{vmatrix} = \hat{j}-\hat{k}$

$= (z-y)\hat{i} - (z-x)\hat{j} + (y-x)\hat{k} = \hat{j}-\hat{k}$

$z-y = 0$ ①

$z-x = 1$ ②

$y-x = -1$ ③

① − ② $\Rightarrow x-y = -1$

$x-y = 1$

$\vec{a}\cdot\vec{c} = 3$

$x+y+z = 3$

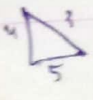
$z = 2$

Q.3 $|\vec{a}| = 10$ $|\vec{b}| = 2$ $\vec{a}\cdot\vec{b} = 12$ then find $|\vec{a}\times\vec{b}|$

$\vec{a}\cdot\vec{b} = |a||b|\cos\theta$

$12 = 20\cos\theta$

$\cos\theta = \frac{12}{20} = \frac{3}{5}$



$\vec{a}\times\vec{b} = |a||b|\sin\theta$

$a\times b = 20\sin\frac{3}{4}$

$a\times b =$

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Ans:- Let $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$

$$(a \times \hat{i}) = \begin{vmatrix} i & j & k \\ x & y & z \\ 1 & 0 & 0 \end{vmatrix} = (z\hat{j} - y\hat{k})$$

$$|a \times i|^2 = |z\hat{j} - y\hat{k}|^2$$

$$= (z\hat{j} - y\hat{k}) \cdot (z\hat{j} - y\hat{k}) \quad \because [|\vec{a}|^2 = \vec{a} \cdot \vec{a}]$$

$$= z^2 + y^2 \quad - (1)$$

$$2|\vec{a}|^2 =$$

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6/11/17 Chapter-11

3-D

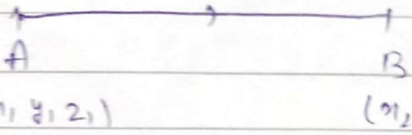
10 Marks

6+4.

Excessica - 11.1

* Direction Ratio and Direction Cosine :

: This is the revision of Ex-10.2 :



DR ⇒

$a = x_2 - x_1$

$b = y_2 - y_1$

$c = z_2 - z_1$

DC ⇒ $l = \cos\alpha$

$m = \cos\beta$

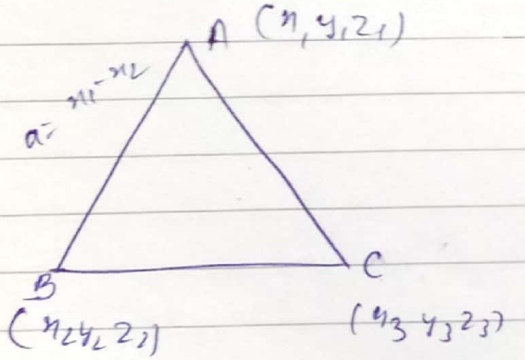
$n = \cos\gamma$

$l^2 + m^2 + n^2 = 1$

$l = \frac{a}{\sqrt{a^2+b^2+c^2}}$ $\quad n = \frac{c}{\quad}$

$m = \frac{b}{\quad}$

*



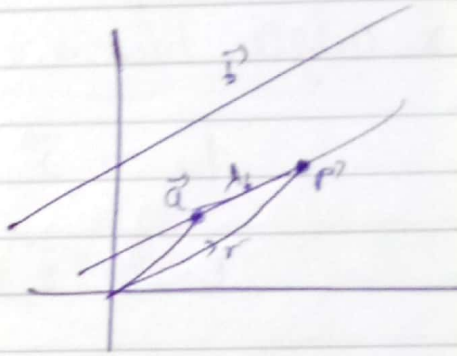
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Ex 11.2 and 11.3
St. line Plane

11.2

* Straight line in 3D :-

(1) eq^n of a line which passing through a point $\vec{a}(x_1, y_1, z_1)$ and Parallel to the vector



(1) (V) $\vec{r} = \vec{a} + \lambda\vec{b}$

(C) $\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$

(V) $\vec{r} = (\hat{i} - \hat{j} + 5\hat{k}) + \lambda(4\hat{i} - 3\hat{j} + 6\hat{k})$

(C) $\frac{x-1}{4} = \frac{y+1}{-3} = \frac{z-5}{6}$

Ex: $\frac{x+1}{-3} = \frac{y+2}{5} = \frac{z-3}{6}$

$\vec{r} = (-\hat{i} - 2\hat{j} + 3\hat{k}) + \lambda(-3\hat{i} + 5\hat{j} + 6\hat{k})$

Ex: $2x = 5y = 6z$

$\frac{x-0}{1/2} = \frac{y-0}{1/5} = \frac{z-0}{1/6}$

$\vec{r} = (0\hat{i} + 0\hat{j} + 0\hat{k}) + \lambda(\frac{1}{2}\hat{i} + \frac{1}{3}\hat{j} + \frac{1}{6}\hat{k})$

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* Two Point form a line :

$\vec{a}$ (x_1, y_1, z_1) $\vec{b}$ (x_2, y_2, z_2)

(v) $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$

(c) $\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$

* angle b/w two lines :

$L_1 \Rightarrow \vec{r} = \vec{a_1} + \lambda(\vec{b_1})$

$L_2 \Rightarrow \vec{r} = \vec{a_2} + \mu(\vec{b_2})$

$$\cos\theta = \frac{\vec{b_1} \cdot \vec{b_2}}{|b_1||b_2|}$$

* (c) $\Rightarrow$ $L_1 = \frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$

$L_2 \Rightarrow \frac{x - x_2}{a_2} = \frac{y - y_2}{b_2} = \frac{z - z_2}{c_2}$

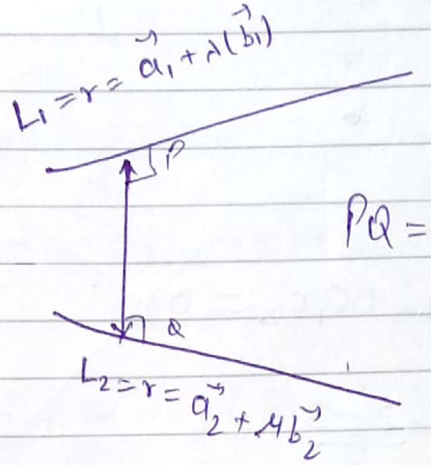
$\cos\theta =$

$L_1 \parallel L_2 = \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

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$L_1 \perp^r L_2 \Rightarrow a_1a_2 + b_1b_2 + c_1c_2 = 0$

* Shortest distance b/w two skew lines :- (neither parallel nor Intersect. non-Coplanar)



$$PQ = \left| \frac{(\vec{b_1} \times \vec{b_2}) \cdot (\vec{a_2} - \vec{a_1})}{|\vec{b_1} \times \vec{b_2}|} \right|$$

* dist. b/w two Parallel line :-

$L_1 = r = \vec{a_1} + \lambda\vec{b}$

$L_2 = \vec{r} = \vec{a_2} + \mu\vec{b}$

$$S.D = \left| \frac{|\vec{b} \times (a_2 - a_1)|}{|\vec{b}|} \right|$$

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11.2 Ques: 6, 7, 11(i), 12, 15, 17

Q.12

$\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2}$

$\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-2}{5}$

means both like lr.

$a_1a_2 + b_1b_2 + c_1c_2 = 0$

$L_1 = \frac{x-1}{-3} = \frac{(y-2)}{\frac{2p}{7}} = \frac{z-3}{2}$

$a_1 \quad a_2 \quad a_3$

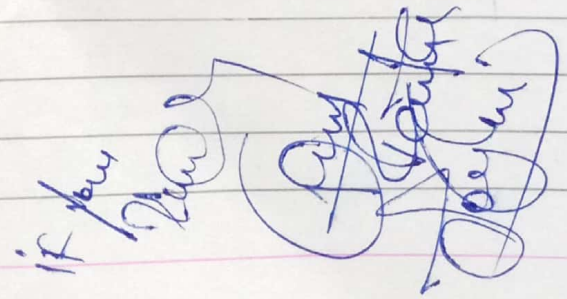
$L_2 = \frac{x-1}{-\left(\frac{3p}{7}\right)} = \frac{y-5}{1} = \frac{3-6}{-5}$

$b_1 \quad b_2 \quad c_2$

$a_1a_2 + b_1b_2 + c_1c_2 = 0$

$-3 \times -\left(\frac{3p}{7}\right) + \frac{2p}{7} \times 1 + 2 \times -5$

$3 \times \frac{3p}{7}$



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(15)

$$L_1 = \frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$$

$$L_2 = \frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$$

$$L_1 = r = (-\hat{i} - \hat{j} - k) + \lambda(7\hat{i} - 6\hat{j} + k)$$
$$\vec{a_1} \qquad\qquad \vec{b_1}$$

$$L_2 = r = (3\hat{i} + 5\hat{j} + 7k) + \mu(\hat{i} - 2\hat{j} + k)$$
$$\vec{a_2} \qquad\qquad \vec{b_2}$$

$$S.D = \left| \frac{(\vec{b_1} \times \vec{b_2}) \cdot (a_2 - a_1)}{|\vec{b_1} \times \vec{b_2}|} \right|$$

(i) $\vec{b_1} \times \vec{b_2} = \begin{vmatrix} i & j & k \\ 7 & -6 & 1 \\ 1 & -2 & 1 \end{vmatrix} = -4\hat{i} - 6\hat{j} + 8k$

(ii) $|\vec{b_1} \times \vec{b_2}| = \sqrt{16 + 36 + 64} = \sqrt{116}$

(iii) $\vec{a_2} - \vec{a_1} = 4\hat{i} + 6\hat{j} + 8k$

(iv) $$S.D = \left| \frac{(\vec{b_1} \times \vec{b_2}) \cdot (a_2 - a_1)}{|\vec{b_1} \times \vec{b_2}|} \right|$$

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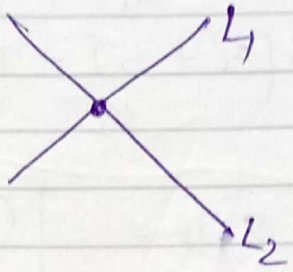
Q.17 Do itself (H.W)

Q. $L_1 = \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$

$L_2 = \frac{x-4}{5} = \frac{y-1}{2} = \frac{z}{1}$

SD = 0

Show that given two line are Intersect. also find the intersection point.



$L_1 = \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda$

$L_2 = \frac{x-4}{5} = \frac{y-1}{2} = \frac{z}{1} = \mu$

Let $P(2\lambda+1, 3\lambda+2, 4\lambda+3)$

$Q(5\mu+4, 2\mu+1, \mu)$

If L_1 & L_2 intersect each other then ~~both considered~~ Point P considered with Q.

$2\lambda + 1 = 5\mu + 4 \quad -(1)$

$3\lambda + 2 = 2\mu + 1 \quad -(2)$

$4\lambda + 3 = \mu \quad -(3)$

$\lambda = -1$

$2\lambda + 1 = 5\mu + 4$ $\quad \mu = -1$ $\times 2$

$-\ 4\lambda + 3 = \mu$

$\circ\ -1 = 9\mu + 8$

$-9 = 9\mu$

$\mu = -1$

from eq^n (1) and (2)

$\lambda = \mu = -1$

Put in eq^4 (3)

LHS = RHL

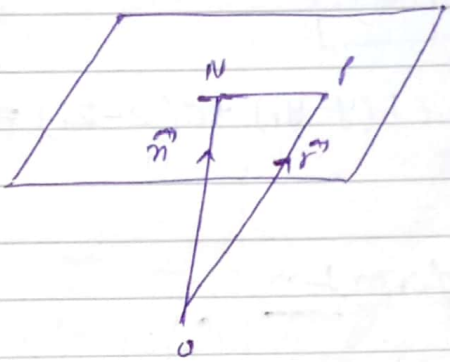
Intersection point = $(-1, -1, -1)$.

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Exercise - 11.3

Plane :

1) normal form of a plane :



(V) (I) $\vec{r} \cdot \vec{n} = q$

$\vec{r} \left(\frac{\vec{n}}{|\vec{n}|}\right) = \left(\frac{q}{|\vec{n}|}\right)$

(2) $\boxed{\vec{r} \cdot \hat{n} = p}$ ← ⊥ dist from origin

$\Rightarrow ax + by + cz = d$

$\frac{a}{\sqrt{a^2+b^2+c^2}} x + by + cz = d$

$\frac{a}{\sqrt{a^2+b^2+c^2}} = L$

$lx + my + nz = p.$

Ex: $\vec{r} \cdot (2\hat{i} + 3\hat{j} + 5\hat{k}) = 7$

$\vec{r} \cdot \frac{(2\hat{i} + 3\hat{j} + 5\hat{k})}{\sqrt{2^2+3^2+5^2}} = \frac{7}{\sqrt{2^2+3^2+5^2}}$ ← Perpendicular dist

$\vec{r} \cdot \hat{n} = p$

Ex: $2x + 4y + 7z = 0$

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(2) One point form :-

$\vec{a}(x_1, y_1, z_1)$

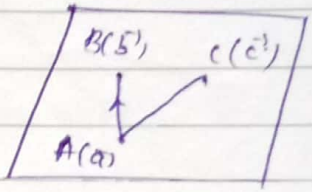
a = Passing Point
$\vec{n}$ = plane normal.

(V) $(\vec{r} - \vec{a}) \cdot \vec{n} = 0$ → Imp.

$\Rightarrow \vec{r} \cdot \vec{n} = (\vec{a} \cdot \vec{n})$

(C) = $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

(3) Three point form :-



(V) $(\vec{r} - \vec{a}) \cdot (\vec{AB} \times \vec{AC}) = 0$

$(\vec{r} - \vec{a}) \cdot [(\vec{b} - \vec{a}) \times (\vec{c} - \vec{a})] = 0$

(C)

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

$A(x_1, y_1, z_1)$ $B(x_2, y_2, z_2)$ $C(x_3, y_3, z_3)$

(V) $\Rightarrow \vec{r} \cdot (2\hat{i} + 3\hat{j} - 6\hat{k}) = 10$

(C) = $2x + 3y - 6z = 10$

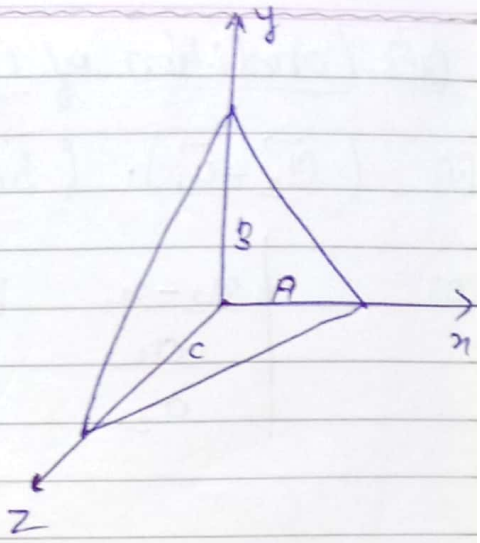
(V) $\vec{r} \cdot (2\hat{i} + 3\hat{j} - 6\hat{k}) = 10$.

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$\frac{x}{a} + \frac{y}{b} =$

(4) Intercept form of a plane :-

$$\frac{x}{A} + \frac{y}{B} + \frac{z}{C} = 1$$



$$\Rightarrow 2x + 4y + 6z = 12$$

$$\frac{2}{12}x + \frac{4y}{12} + \frac{6z}{12} = 1$$

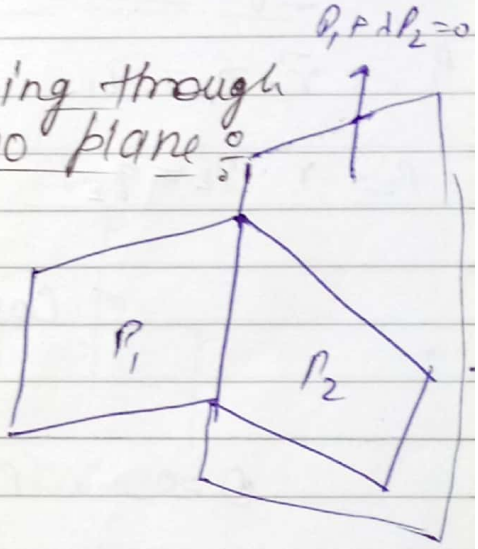
Very Important

(5)

90%

Equation of a plane which passing through intersection line of two plane :-

$$P_1 + \lambda P_2 = 0$$

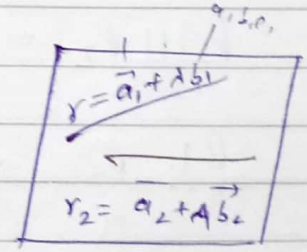


(6) equation of a plane which containing two line :-

(V) $(\vec{r} - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = 0$

(C)
$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

(7)



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(7) Condition of Co-Planarity of two lines:

(v) $(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2}) = 0$

(c)
$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

(8) angle b/w two Plane:

P_1 $\quad \vec{r} \cdot n_1 = q_1$

P_2 $\quad \vec{r} \cdot n_2 = q_2$

$$\cos\theta = \frac{n_1 \cdot n_2}{|\vec{n_1}|\,|\vec{n_2}|}$$

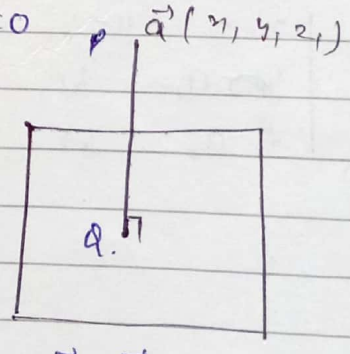
$\theta = 0 \quad P_1 \parallel P_2$

$\theta = 90^\circ \quad P_1 \perp P_2$

$P_1 \parallel P_2 \Rightarrow \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$P_1 \perp P_2 \Rightarrow \quad a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$

(9) Perpendicular dist. from one point:



$\vec{r} \cdot \vec{n} = q$

$ax + by + cz = d$

(c)
$$PQ = \left| \frac{ax_1 + by_1 + cz_1 - d}{\sqrt{a^2 + b^2 + c^2}} \right|$$

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(v) $PQ = \left| \frac{\vec{a}\cdot\vec{n} - q}{|\vec{n}|} \right|$

(iv) Angle b/w line and plane :

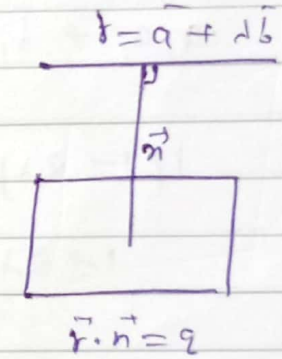
$L = \vec{r} = \vec{a} + \lambda\vec{b}$

$P \Rightarrow \vec{r}\cdot\vec{n} = q$

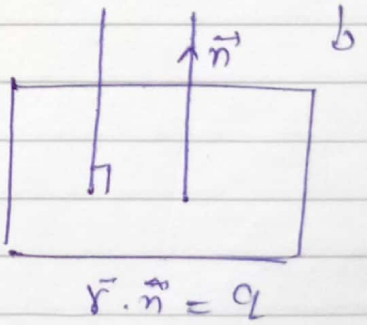
$$q \sin\phi = \frac{\vec{b}\cdot\vec{n}}{|\vec{b}||\vec{n}|}$$

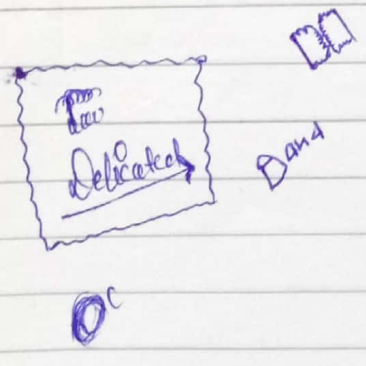
Plane and line are parallel :

(i) $\vec{b}\cdot\vec{n} = 0$



(ii) $\vec{b} = \vec{n}$





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11.3)

Q.2 $\vec{r} \cdot \frac{\vec{n}}{|\vec{n}|} = 7$

Q.11 $P_1 = x + y + z - 1 = 0 \Rightarrow \vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 1$ ($\vec{n}_1$)

$P_2 = 2x + 3y + 4z - 5 = 0 \quad \vec{r} \cdot (2\hat{i} + 3\hat{j} + 4\hat{k}) = 5$ ($\vec{n}_2$)

$P_3 = \underset{a_2}{x} - \underset{b_2}{y} + \underset{c_2}{z} = 0$

$\vec{n_1} \cdot \vec{n_2} = 0$

Req - $P_1 + \lambda P_2 = 0$

$= x(1 + 2\lambda) + y(1 + 3\lambda) + 3(1 + 4\lambda) - 1 - 5\lambda = 0 \quad - (1)$

(a_1, b_1, c_1)

plane 1 is $\perp$ to P_3

$a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$

$1(1 + 2\lambda) + (-1)(1 + 3\lambda) + 1(1 + 4\lambda) = 0$

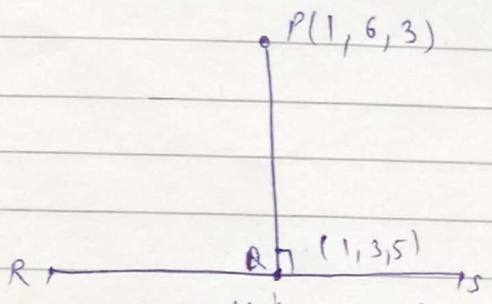
$1 + 2\lambda - 1 - 3\lambda + 1 + 4\lambda = 0$

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Q. Find the foot of $\perp^r$ drawn from the point (1,6,3) on the line $L = \frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ also find its distance from point

eqⁿ of $\perp^r$ and Image of a point

$\hat{i} + 2\hat{j} + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$. P(1,6,3)



$\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$

$P'(\alpha, \beta, \gamma)$

$\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3} = \lambda$

$a_1 \quad b_2 \quad c_3$

$Q(\lambda, 2\lambda+1, 3\lambda+2)$

$a_1a_2 + b_1b_2 + c_1c_2 = 0$

$\lambda = ?$

$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$

eqⁿ ration of PQ.

$PQ = \sqrt{(x_2-x_1)^2 + \dots}$

$=$

$(\lambda-1, 2\lambda+1-6, 3\lambda+2-3\lambda)$

Line $PQ \perp RS$.

$(\lambda-1)1 + (2\lambda+1-6)2 + (3\lambda+2-3)3$

$\lambda - 1 + 4\lambda - 10 + 9\lambda - 3 = 0$

$\Rightarrow 14\lambda = 14$

$\lambda = 1$

$(1, 3, 5)$

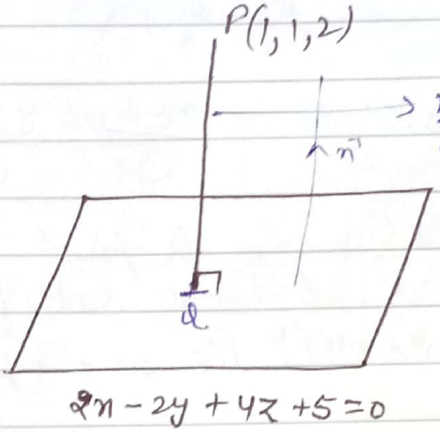
foot of $\perp^r$.

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Q is a mid point of PP'

$\left(1 = \frac{1+\alpha}{2} ; 3 = \frac{6+\beta}{2}, 5 = \frac{3+\gamma}{2}\right)$ Image.

Q.



Dr. of $\frac{x-1}{2} = \frac{y-1}{-2} = \frac{z-2}{4} = \lambda$

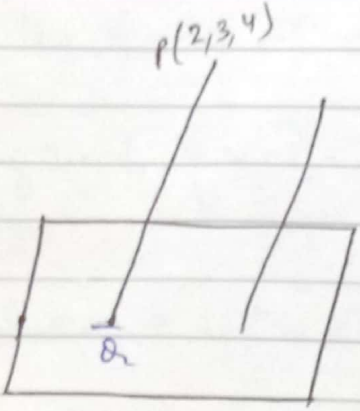
satisfy Q. in plane.

$\lambda = -\frac{13}{24}$

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eq^n of line $r = a + \lambda b$

Q.



$\frac{x+3}{3} = \frac{y-2}{6} = \frac{z}{2}$

$3x + 2y + 2z + 5 = 0$

eq^n of PQ

$\frac{x-2}{3} = \frac{y-3}{6} = \frac{z-4}{2} \Rightarrow$

Q find
Q satisfy plane.

point

$Q(3\lambda+2, 6\lambda+3, 2\lambda+4)$ lie on the plane

$3(3\lambda+2) + 2(6\lambda+3) + 2(2\lambda+4) + 5 = 0$

$9\lambda + 6 + 12\lambda + 6 + 4\lambda + 8 + 5 = 0$

$25\lambda = -25$

$\lambda = -1$

$Q(-1, -3, 2)$.

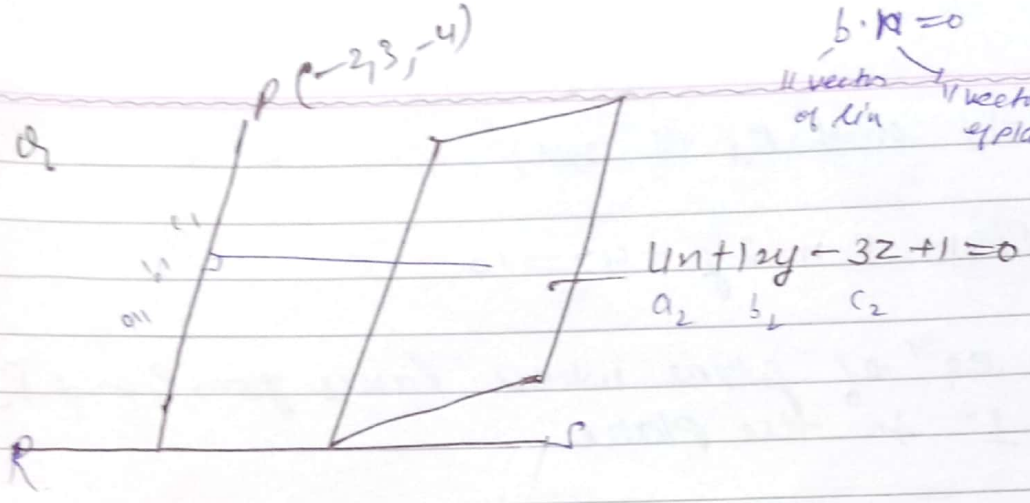
dist of line $PQ = \sqrt{(-1-2)^2 + (-3-3)^2 + (2-4)^2}$

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If a line is ⊥el to Plan

$\vec{b} \cdot \vec{n} = 0$

$\vec{b}$ ∥ vector of line, $\vec{n}$ ∥ to vector of plane



$$\frac{x+2}{3} = \frac{2y+3}{4} = \frac{3z+4}{5} = \lambda$$

D.R. $Q(3\lambda-2, \frac{4\lambda-3}{2}, \frac{5\lambda-4}{3})$

D.R of line PQ: $(3\lambda-2+2, \frac{4\lambda-3}{2}-3, \frac{3\lambda+4}{5}+4)$

a_1 b_1 c_1

$$\left(3\lambda, \frac{6-4\lambda-3}{2}, \frac{20+3z+4}{5}\right)$$

$$\left(3\lambda \quad \frac{3-4\lambda}{2}, \quad \frac{24+3\lambda}{5}\right)$$

$$4(3\lambda) + 12\left(\frac{3-4\lambda}{2}\right) - 3\left(\frac{24+3\lambda}{5}\right) + 1 = 0$$

$$12\lambda + 6(3-4\lambda) - 3\left(\frac{24+3\lambda}{5}\right) + 1 = 0$$

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Q. $P_1(2, 1, -1)$ and $P_2(-1, 3, 4)$

Plane : $x - 2y + 4z = 10$

find the eq^n of plane whose passes from P_1 and P_2 and ⊥^{r} to the plane.

$$x - 2y + 4z = 10$$

$$\frac{x-2}{-1} = \frac{y-1}{3} = \frac{z+1}{4} = \lambda$$

$$(-\lambda+2,\ 3\lambda+1,\ 4\lambda-1)$$

$$-\lambda + 2 - 2(3\lambda+1) + 4(4\lambda-1) = 0$$
$$-\lambda + 2 - 6\lambda - 2 + 16\lambda - 4 = 0$$
$$9\lambda = 4$$

$x-2$

18 17 4

D.r of normal vector of plane

Ans: $a(x-x_1) + b(y-y_1) + c(z-z_1) = 0$

$P_1(2, 1, -1)$

$$a(x-2) + b(y-1) + c(z+1) = 0 \quad \text{---(8)}$$

$P_2(-1, 3, 4)$

Point P_2 in a plane

So,

$$(-1-2)a + (3-1)b + (4+1)c = 0$$
$$-3a + 2b + 5c = 0 \quad \text{---(1)}$$

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Required Plane is ⊥r to the given plane

$$a-2b+4c=0 \quad -(2)$$

$$-3a+2b+5c=0 \quad -(1)$$
$$a-2b+4a=0$$

$$\frac{a}{8+10} = \frac{-b}{-12-5} = \frac{c}{6-2} = \lambda.$$

Put value of a, b, c in eq^n (3).

Q. Prob.

$P(1,1,1)$

Line $\vec{r} = (-3i+j+5k) + \lambda(3i-j-5k)$

Also $\vec{r} = (-i+2j+5k) + \lambda(i-2j-5k)$.

Q. find the eq^n of a plane passing from a point (1,1,1) and containing the line

$$\vec{r} = (-3i+j+5k) + \lambda(3i-j-5k)$$

also show that the plane containing the line

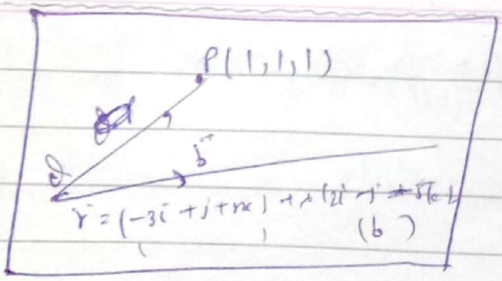
$$\vec{r} = (-i+2j+5k) + \lambda(i-2j-5k).$$

$$a(x-x_1) + b(y-y_1) + c(z-z_1) = 0$$
$$a(x-1) + b(y-1) + c(z-1) = 0$$

$$-3(x-1) + 1(y-1) + 5(z-1) = 0$$
$$-3x+3+y-1+5z-1=0$$
$$-3x+y+5z=-1$$
$$3x-y-5z-1=0.$$

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Sol



P (A) position vector (A)

$\vec{b} \cdot \vec{n} = 0$

$(\vec{r} - \vec{a}) \cdot \vec{n} = 0$

$[\vec{r} - (i + j + k)] \cdot \vec{n} = 0$ — (1)

normal vector of Req. Plane

$\vec{n} \parallel (\vec{QP} \times \vec{b})$

$(\vec{OP} - \vec{OQ} \times \vec{b})$

Q. P_1 ; $\vec{r} \cdot (2i - 3j + 6k) = 5$

P_2 ; $\vec{r} \cdot (6i - 9j + 18k) + 20 = 0$

dist = ?

- Parallel Plane

$$\frac{|d_1| \cdot |d_2|}{\sqrt{a^2 + b^2 + c^2}}$$

|| plane dist $= \dfrac{d_1 - d_2}{\sqrt{a^2 + b^2 + c^2}}$

Q. Find the eq^n of the two lines through the origin which intersect the line $\frac{x-3}{2} = \frac{y-3}{1} = \frac{z}{1}$ at angle of $\frac{\pi}{3}$ each

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Chapter – 12

6 Mark

L.P.P (Linear Programming Problem)

12.1

* Mathematically form of L.P.P :

$z = ax + by$

(objective function)

Max

Min.

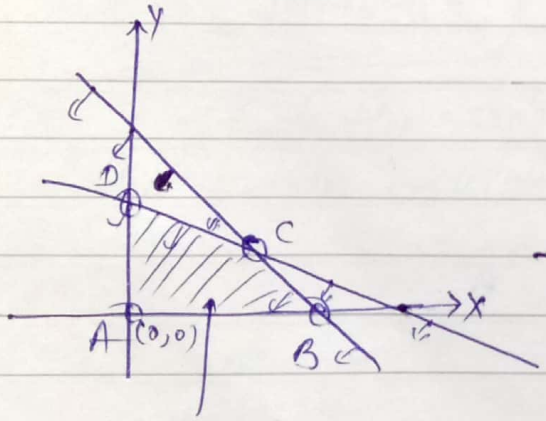
Subject to Constants

$a_1x + b_1y \geq$ OR $\leq c_1$

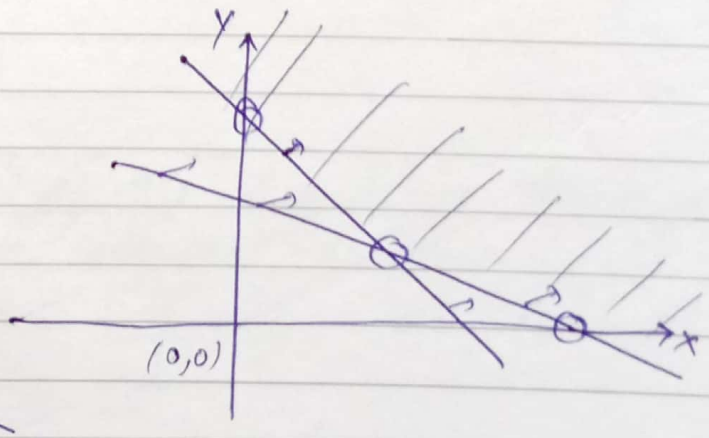
$a_2x + b_2y \geq$ OR $\leq c_2$

non-negative value. $\{ \quad x, y \geq 0$

* Corner Method :



Bounded fessiable Region



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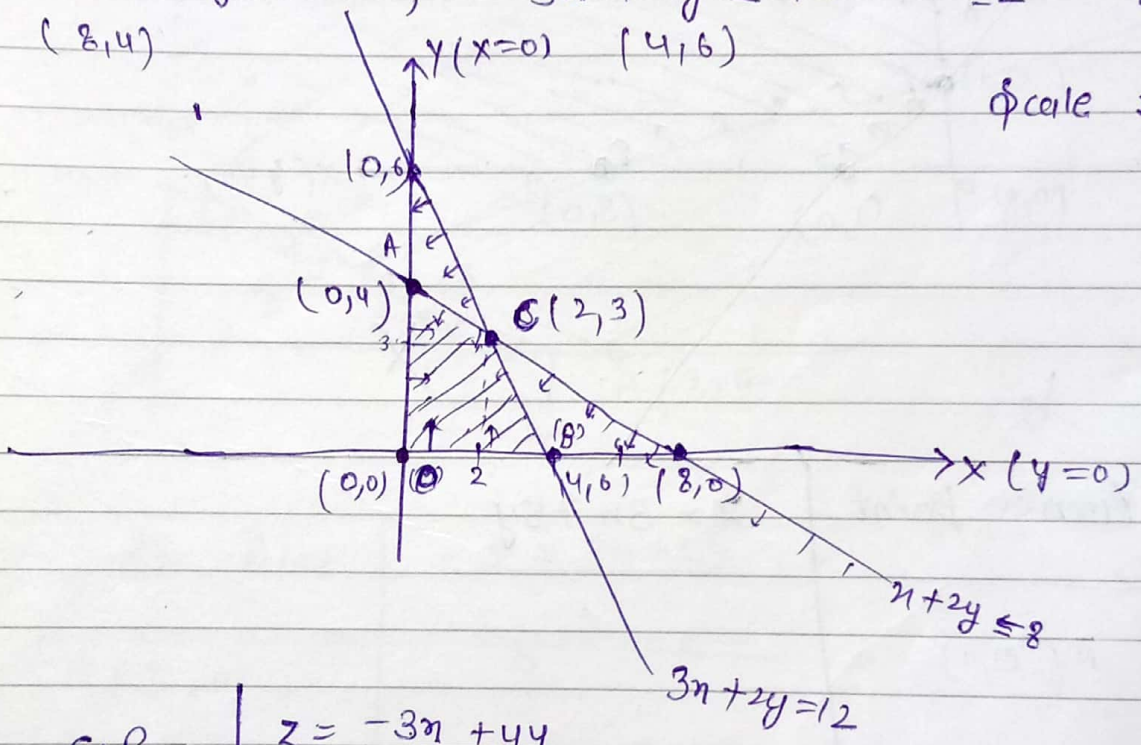
12.2 ⇒ 2

C.P	$Z = ax + by$
A	↓
B	
C	
D	

2.12.12

(2) $Z = -3x + 4y$

$x + 2y \leq 8$, $3x + 2y \leq 12$ $x \geq 0$ $y \geq 0$



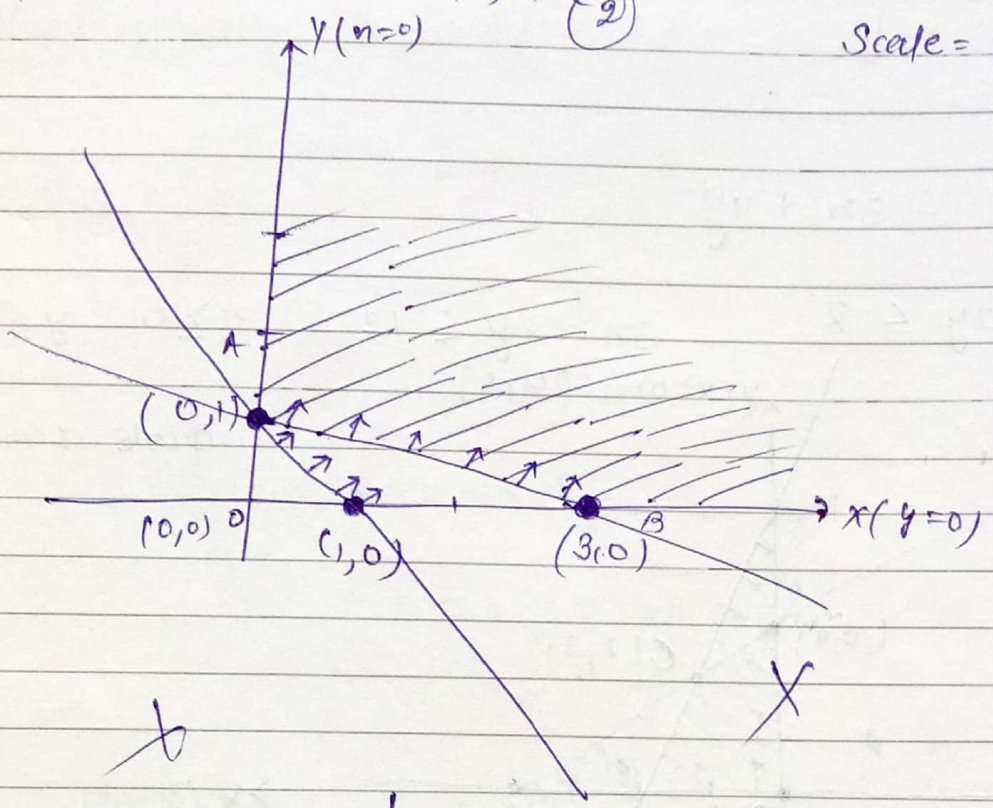
C.P	$Z = -3x + 4y$
A (0,4)	= 16
O (0,0)	0
B (4,0)	−12
C (2,3)	6

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Q.y $z = 3x + 5y$

$x + 3y \geq 3$, $x + y \geq 1$, $x, y \geq 0$

(3, 1) (1, 1) (2)



Corner point	$z = 3x + 5y$
A (0, 1)	
B (3, 0)	

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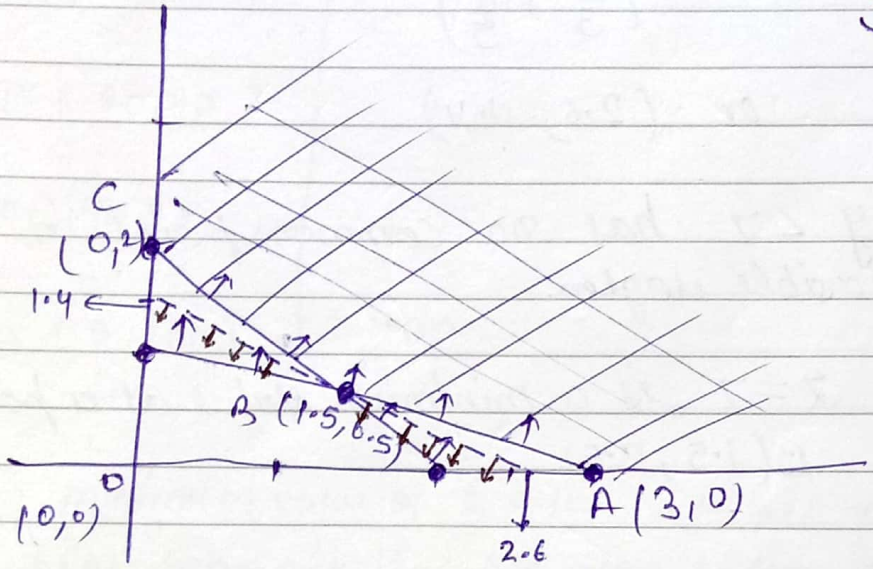
Ans: $Z = 3x + 5y$

$x + 3y \geq 3 \quad (3,1)$

$x + y \geq 2 \quad (2,2)$

$x, y \geq 0$

scale 2cm = 1 (unit)



Cor. Point	$Z = 3x + 5y$
A (3, 0)	9
B (1.5, 0.5)	4.5 + 2.5 = 7
C (0, 2)	10.

minimum

for confirmation of point B is a unique point where Z give minimum value 7.

$3x + 5y < 7$

Draw the inequality $3x + 5y < 7$

$\left(\frac{7}{3}, \frac{7}{5}\right)$

or $(2.6, 1.4)$

$3x + 5y < 7$ has no common point in unbounded feasible region

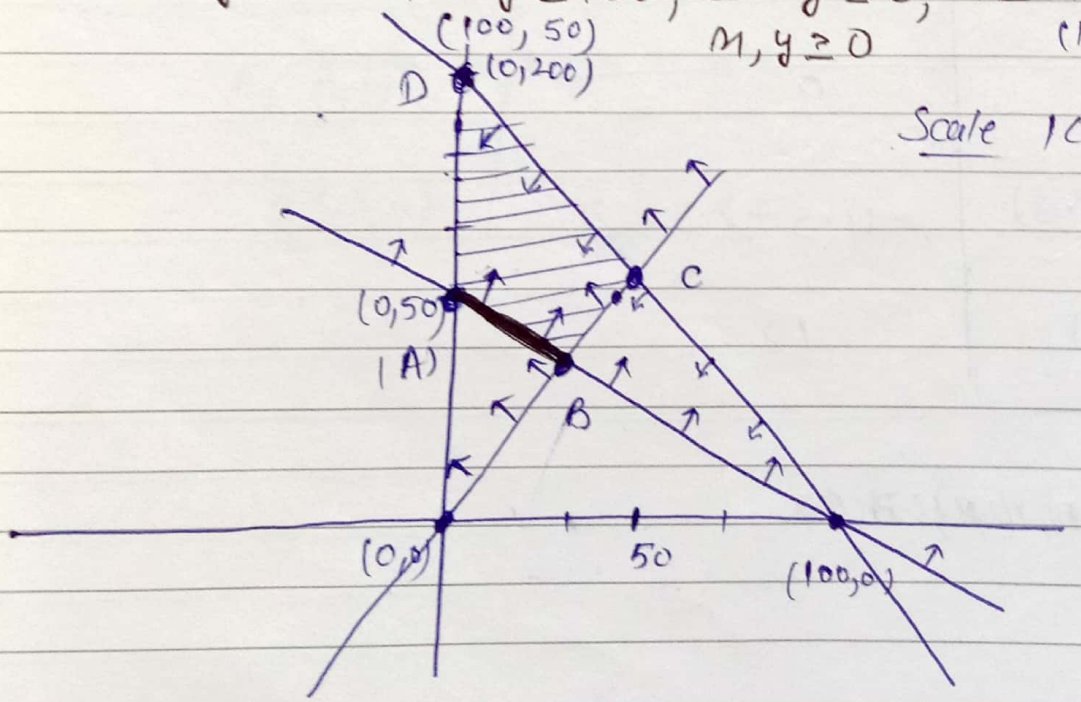
Hence $Z = 7$ is a minimum value at a point $B(1.5, 0.5)$

Que: 8

$Z = x + 2y$ (0, 0)

Subject $x + 2y \geq 100$, $2x - y \leq 0$, $2x + y \leq 200$ (100, 200)

$x, y \geq 0$



Scale 1 cm = 50 unit

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No comm

Q. 7, 6.

$2x - y = 0$

$y = 2x + 0$

$2x - y = 0$)×2

$x + 2y = 100$

$4x - 2y = 0$

$5x = 100$

$x = 20$

$x = 50, \quad y = 100$

C.P	$Z = x + 2y$
A (0, 50)	100
B (20, 40)	100
C(50, 100)	250
D (0, 200)	400

minimum value of $Z = 100$ at all the point on the line segment joining the point A and B.

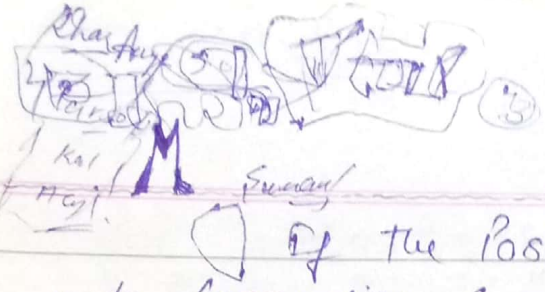
7, 6, 5, 4, 3, 2, 1

7 to 1

1 cm to 1 mm

0.01 m to 2 m

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of the position
under conservation. Curve is above
If some portion of the curve is above 1-axis of some (a, b)
& the
x-axis

$$\text{Area} = \int_c^d x\,dy = \int_c^d f(y)\,dy$$

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12 Marks

Chapter-7 Integrals

$\int x^n dx = \frac{x^{n+1}}{n+1} + C$

$\int \frac{1}{x} = \log|x| + C$

$\int \frac{1}{x^2} = -\frac{1}{|x|} + C$

$\int \frac{1}{x^3} = \frac{-1}{2x^2} + C$

$\int \frac{1}{\sqrt{x}} dx = 2\sqrt{x} + C$

$\int \sin x dx = -\cos x$

$\int \cos x dx = \sin x$

$\int \sec^2 x dx = \tan x$

$\int \operatorname{cosec}^2 x dx = -\cot x$

$\int \sec x \tan x dx = \sec x$

$\int \operatorname{cosec} x \cot x dx = -\operatorname{cosec} x$

$\int e^x \cdot dx = e^x + C$

$\int e^2 \cdot dx = x \cdot e^2 + C$

$\int a^x dx = \frac{a^x}{\log a}$

$\int 3^x dx = \frac{3^x}{\log 3} + C$

$\frac{1}{x^3} = x^{-2}$

$= \frac{x^{-2}}{-2}$

$= -\frac{1}{2x^2}$

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$$\int \tan x\,dx = \log \sec x + C$$

$$\int \cot x\,dx = \log \sin x + C$$

$$\int \sec x\,dx = \log|\sec x + \tan x|$$

OR $\log\left|\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right|$

$$\int \operatorname{cosec} x\,dx = \log|\operatorname{cosec} x - \cot x|$$

OR $\log\left|\tan \frac{x}{2}\right| + C$

$$\int \sin^2 x = \int \frac{1-\cos 2x}{2}\,dx = \frac{1}{2}\left[x - \frac{\sin 2x}{2}\right]$$

$$\int \cos^2 x = \int \frac{1+\cos 2x}{2}\,dx = \frac{1}{2}\left[x + \frac{\sin 2x}{2}\right]$$

$$\int \tan^2 x\,dx = \int (\sec^2 x - 1) = \tan x - x + C$$

$$\int \cot^2 x = \int (\operatorname{cosec}^2 x - 1) = \cot x - x + C$$

$$\Rightarrow \int \frac{1}{\sqrt{1-x^2}} = \sin^{-1} x + C$$

$$\int \frac{-1}{\sqrt{1-x^2}} = \cos^{-1} x + C$$

$$\int \frac{1}{1+x^2}\,dx = \tan^{-1}$$

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$$\int \frac{-1}{1+x^2}dx = \cot^{-1}x \qquad \int \frac{1}{x^2-a^2} = \frac{1}{2a}\log\frac{x-a}{x+a}$$

$$\int \frac{1}{x\sqrt{x^2-1}} = \sec^{-1}x + C$$

$$\int \frac{-1}{x\sqrt{x^2-1}} = \operatorname{cosec}^{-1}x + C$$

$$\int \sqrt{1 \pm \sin 2x} = \int \cos x \pm \sin x$$

$$\int \sqrt{1+\cos 2x} = \sqrt{2}\int \cos x$$

$$\int \sqrt{1-\cos 2x} = \sqrt{2}\int \sin x$$

$$\int \frac{1 \pm \tan x}{1 \pm \tan x} = \int \tan\left(\frac{\pi}{4} \pm x\right) \qquad -\log\sec\left(\frac{\pi}{4}-x\right)$$

Que. $\int \frac{x^3 - x^2 + x - 1}{x-1}dx.$ $\frac{x^2(x-1)+1(x-1)}{x-1} = \int x^2+1$

Q. $\int \frac{x^4 - x^3 + x^2 + 1}{x^2-1}dx$

$$\int x^2 - x + 2 \; \frac{-x+3}{x^2-1}$$

$x^2-1 \;\big)\; x^4 - x^3 + x^2 + 1 \;\big(\; x^2+1$

$x^4 \mp x^2$

$x + x^2 + 1$

$x^2 -$

$3x$

$\frac{1}{x^2-1}$

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Q. $\int \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2$

$\int \left(x + \frac{1}{x} + 2\right)$

$\frac{x^2}{2} + \log x + 2x$

Q. $\int \frac{x^3 + x^2 + 1}{x}$

$\int \frac{x^3}{x} + \frac{x^2}{x} + \frac{1}{x}$

$\int x^3 \quad \frac{x^3}{3} + \frac{x^2}{2} + \log x.$

Q. $\int (\sin x + \cos x)^2$

$\int (1 + \sin 2x) = x - \frac{\cos 2x}{2} + c$

$\frac{1 - \cos 2x}{2}$

Q. $\int \frac{1}{\sin^2 x \cdot \cos^2 x}$

Method-1 $4\int \frac{1}{2^2 \sin^2 x \cdot \cos^2 x} = 4\int \frac{1}{\sin^2 2x}$

$= 4\int \operatorname{cosec}^2 2x = 4 \times -\frac{\cot 2x}{2}$

$= -2 \cot 2x$

M-2 $\int \frac{1}{\sin^2 x \cos^2 x}$

$= \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} = \int \sec^2 x + \int \operatorname{cosec}^2 x$

$\tan x - \cot x + c.$

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Prob.

Que: $\int \frac{1}{\sin x + \cos x}$

$$\frac{1}{\sqrt{2}} \int \frac{1}{\left(\frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x\right)}$$

$$\frac{1}{\sqrt{2}} \int \frac{1}{\sin\left(x + \frac{\pi}{4}\right)} dx$$

$$\frac{1}{\sqrt{2}} \int \operatorname{cosec}\left(x + \frac{\pi}{4}\right) dx$$

or $\frac{1}{\sqrt{2}} \log \left| \tan\left(\frac{x}{2} + \frac{\pi}{8}\right) \right| + C$

Method-2 $I = \int \frac{1}{\sin x + \cos x}$

$$I = \int \frac{1}{\frac{2\tan x/2}{1+\tan^2 x/2} + \frac{1-\tan^2 x/2}{1+\tan^2 x/2}}$$

Not Complete.

Learn 1st step Method

Q.2 (i) $\int \frac{1}{\sin(x-a)\sin(x-b)} dx.$

(ii) $\int \frac{1}{\cos(x-a)\cos(x-b)} dx$

(iii) $\int \frac{1}{\sin(x-a)\cos(x-b)} dx$

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$$\int \frac{f'(x)}{f(x)}\,dx = \log|f(x)|$$

Learn

Ans (i) ⊕ divide and multiply

$\frac{1}{\sin(a-b)}$ (for same)

$\frac{1}{\cos(a-b)}$ (for diff)

Ans (i)

$$\frac{1}{\sin(a-b)} \int \frac{\sin(a-b)}{\sin(x-a)\cdot\sin(x-b)}\,dx$$

$$\frac{1}{\sin(a-b)} \int \frac{\sin[(x-b)-(x-a)]}{\text{---}}$$

$$\frac{1}{\sin(a-b)} \int \frac{\sin(x-b)\cos(x-a) - \cos(x-b)\sin(x-a)}{\sin(x-a)\sin(x-b)}$$

$$\frac{1}{\sin(a-b)} \left[\int \left[\cot(x-a) - \cot(x-b)\right] dx\right.$$

$$= \sin(a-b)\left[\log\sin(x-a) - \log\sin(x-b)\right.$$

$$\frac{1}{\sin(a-b)} \log\left|\left(\frac{\sin(x-a)}{\sin(x-b)}\right)\right| + C$$

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(iii) $\frac{1}{\cos(a-b)} \int \frac{\cos(a-b)}{\sin(x-a)\cos(x-b)}$

$\frac{1}{\cos(a-b)} \int \frac{\cos[(x-b)-(x-a)]}{\sin(x-a)\cos(x-b)}$

$\frac{1}{\cos(a-b)} \int$

Imp.

3) $I = \int \frac{1}{1+\tan x}\, dx.$

$\int \frac{1}{1+\frac{\sin x}{\cos x}} = \int \frac{\cos x}{\cos x + \sin x}\, dx.$

Motor cycle ka simple banda mai sin

$\frac{1}{2} \int \frac{2\cos x}{\cos x + \sin x}\, dx.$

$= \frac{1}{2} \int \frac{\cos x + \cos x + \sin x - \sin x}{\cos x + \sin x}$

$= \frac{1}{2} \int \left[1 + \frac{\cos x - \sin x}{\cos x + \sin x} \right] dx$

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$$\left[\because \int \frac{f'(x)}{f(x)} dx = \log f(x) \right]$$

$$= \frac{1}{2} \left[x + \log |\cos x + \sin x| \right] + c.$$

(11) $\int \frac{1}{1 - \cot x}.$

$$\frac{1}{2} \int \frac{2}{1 - \frac{\cos x}{\sin x}} = \frac{1}{2} \int \frac{2 \sin x}{\sin x - \cos x}$$

$$\frac{1}{2} \int \frac{\sin x + \sin x - \cos x + \cos x}{\sin x - \cos x}$$

(12)

$$\frac{1}{2} \int \left[1 + \frac{\sin x + \cos x}{\sin x - \cos x} \right] dx$$

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Learn full formula
7.1 and 7.2

Imp. Make
Q.4

$$\int \frac{\sqrt{\tan x}}{\sin x \cdot \cos x}\, dx$$

$\frac{\sin x}{\cos x}$ $\frac{1}{2}$

$\frac{\sqrt{\sin x}}{\sqrt{\cos x \sin x \cos x}} = 1$

$$\int \frac{\sqrt{\tan x}}{\frac{\sin x}{\cos x} \cdot \cos x \cos x}$$

$\tan x = t$

$\sec^2 x\, dx = dt$

$$= \int \frac{\sec^2 x}{\sqrt{\tan x}}\, dt$$

$$= \int \frac{dt}{\sqrt{t}}$$

$$= 2\sqrt{t}$$

$$= 2\sqrt{\tan x} + C.$$

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* Substitution Method :-

1) $I = \int f(x) \cdot f'(x)\, dx$

2) $I = \int f[g(x)] \cdot g'(x)\, dx$.

3) $I = \int \frac{f'(x)}{f(x)} \cdot dx = \log|f(x)| + c$

(1) put $f(x) = t$

$f'(x)\, dx = dt$

$I = \int t \cdot dt = \frac{t^2}{2} + c$

Q. $I = \int \frac{\log x}{x} \cdot dx$

Put $\log x = t$

$\frac{1}{x} dx = dt$

$\int t \cdot dt = \frac{t^2}{2} + c$

(2) Qu. $\int \frac{3x^2 \sin(\tan^{-1} x^3)}{1 + x^6} dx$

$\int \tan^{-1} x^3 = t$

$\frac{3x^2}{1 + x^6} dx = dt$

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$\sin t \cdot dt$

$-\cos(\tan^{-1} x^3) + C$

Q. $I = \int \frac{\cos x - \sin x}{\cos x + \sin x} \cdot dx$

$I = \int \frac{dt}{t}$

$= \log|t| + C$

$\log|\cos x + \sin x|$

$I = \int \frac{e^{2x} - 1}{e^{2x} + 1} dx$

$\int \frac{e^x - e^{-x}}{e^x + e^{-x}}$

Put $e^x + e^{-x} = t$.

Ex - 7.2

6 to 11 same

Q.10

$I = \int \frac{1}{x - \sqrt{x}} dx.$

$= 2 \int \frac{1}{2\sqrt{x}(\sqrt{x}-1)} dx$

$\sqrt{x} = t$

$\frac{1}{2\sqrt{x}} dx = dt$

$I = \int \frac{1}{\sqrt{x}(\sqrt{x}-1)} dx$

Put $\sqrt{x} - 1 = t$ $\quad \frac{1}{2\sqrt{x}} dx = dt$

$= \int \frac{2dt}{t}$ $\quad \frac{dx}{\sqrt{x}} = 2dt$

$= 2 \log t + C$

$2 \log|\sqrt{x} - 1| + C$ Ans

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Method-2 $I = \int \frac{1}{x - \sqrt{x}}\, dx$ Put $\sqrt{x} = t$

$x = t^2$

$dx = 2t\,dt$

$\int \frac{2t\,dt}{t^2 - 1}$

$2\int \frac{1}{(t-1)}\, dt$

$2 \log|t-1| + C$

$2 \log|\sqrt{x} - 1| + C.$

(11) $\int \frac{x}{\sqrt{x+4}}\, dx.$ Put $\sqrt{x+4} = t$

$x = t$ $\sqrt{x+4} = t$

$\int \frac{t^2 - 4}{t}$

$x + 4 = t^2$

$dx = 2t\,dt.$

$\int \frac{x \cdot 2t\,dt}{t}$

$= \int 2x\,dt = 2\int x\,dt =$

(12) $\int (x^3 - 1)^{1/3} x^5$

$\int (x^3 - 1)^{1/3} x^3 \cdot x^2\, dx.$

Put $x^3 - 1 = t$ $3x^2\,dx = dt.$

$\frac{1}{3} \int t^{1/3} \cdot dt$

$=$

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$\log|\tan\frac{\pi}{5}|$

Q. $I = \int \frac{\cos x}{\sqrt{1+\sin x}}\,dx.$

Prob.

(2) $I = \int \frac{\cos x}{\sqrt{1+\sin x}}$

$$\int \frac{1+\sin x}{\sqrt{1+\sin x}} - \frac{1}{\sqrt{1+\sin x}}$$

$$\int \sqrt{1+\sin x} - \int \frac{1}{\sqrt{1+\sin x}}$$

$$\int \left(\cos\frac{x}{2} + \sin\frac{x}{2}\right) - \frac{1}{\sqrt{2}}\int \frac{1}{\frac{1}{\sqrt{2}}\cos\frac{x}{2} + \frac{1}{\sqrt{2}}\sin\frac{x}{2}}$$

$$-2\int \frac{1}{\frac{1}{\sqrt{2}}\cos\frac{x}{2} + \frac{1}{\sqrt{2}}\sin\frac{x}{2}}$$

$$-\frac{1}{2}\int \frac{1}{\sin\left(\frac{\pi}{2}+\frac{x}{2}\right)} = -\frac{1}{2}\int \operatorname{cosec}\left(\frac{\pi}{2}+\frac{x}{2}\right)$$

$$\frac{-\frac{1}{\sqrt{2}}\log\tan\left(\frac{\pi}{8}+\frac{x}{4}\right)}{\frac{1}{\sqrt{2}}}$$

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Q. $\int \frac{10x^9 + 10^x \log_e 10}{x^{10} + 10^x} dx$

(3b) $I = \int \frac{10x^9 + 10^x \log_e 10}{x^{10} + 10^x} \cdot dx \Rightarrow \int \frac{dt}{t} = \log(t)$

$x^{10} + 10^x = t$

$10x^9 + 10^x \log 10 \, dx = dt$

* 7.3

Integration using trigonometric identities

formulas:

1. $\sin A \cos B = \sin(A+B) + \sin(A-B)$
2. $\cos A \sin B =$
3. $\cos A \cos B = \cos(A+B) + \cos(A-B)$
4. $2 \sin A \sin B = \cos(A-B) - \cos(A+B)$

$\sin^2 x = \frac{1 - \cos 2x}{2}$

$\cos^2 x = \frac{1 + \cos 2x}{2}$

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$$z + z^{-1} = 2\cos\theta$$
$$z - z^{-1} = (2i\sin\theta)$$

Q. $\int \sin 2n \sin 4n \sin 6n \cdot dn$ = $\frac{\sin(2n+4n)}{\sin 2n \cos 4n}$

$\frac{1}{2}\int 2\sin 2n \sin 4n \sin 6n \cdot dn$

$\frac{1}{2}\int (\cos 2n - \cos 6n) \cdot \sin 6n$

$\frac{1}{2}\int \cos 2n \cdot \sin 6n - \cos 6n \sin 6n]$

$\frac{1}{2}\int \left(\frac{2}{2}\cos 2n \cdot \sin 6n\right) - \left(\frac{2}{2}\cos 6n \sin 6n\right)]$

* Based only sinn or cosn.
Power

Q.

(1) even

Q $\int \sin^4 n \cdot dn$ or $\int \cos^4 n \cdot dn$

Ans

Q. $\int \cos^6 n$

$\int \sin^8 n \cdot dn$

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$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

$\cos 3\theta =$

① $\int (\sin^2 x)^2 \cdot dx$

$= \frac{1}{4}\int (1-\cos 2x)^2$

$\frac{1}{4}\int 1+\cos^2 2x - 2\cos 2x$

$\frac{1}{4}\int \left[1 + \left(\frac{1+\cos 4x}{2}\right) - 2\cos 2x\right.$

$\int \cos^4 x \cdot dx$

$\int (\cos^2 x)^2 \cdot dx$

$\frac{1}{4}\int (1-\sin 2x)^2$

$\frac{1}{4}\int 1+\sin^2 2x - 2\sin 2x$

$\frac{1}{4}\int \left[1 + \left(\frac{1+\sin 4x}{2}\right) - 2\sin 2x\right.$

$2\sin^2 x = 1-\cos 2x$

$\sin^2 x = \frac{1-\cos 2x}{2}$

(3) Even - Even Power.

$\int \sin^2 x \cdot \cos^4 x \cdot dx$

$\int \left(\frac{1-\cos 2x}{2}\right) \times \left(\frac{1+\cos 2x}{2}\right)^2$

$\frac{1}{8}\int \sin^2 2x$

$\frac{1}{2}(1-\cos 2x)\left(\frac{1+\cos^2 2x + 2\cos 2x}{2}\right)$

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(3) odd :

$$I = \int \sin^5 x \, dx \quad = \quad I = \int \sin^4 x \cdot (\sin x \, dx)$$

$$= \int (\sin^2 x)^2 \, (-\text{''}\,)$$

$$= \int (1 - \cos^2 x)^2 (\sin x \, dx) = \int (1-t^2)^2 \cdot dt$$

$$- \int (1 + t^4 - 2t^2) \, dt \quad = \quad - \left[t + \frac{t^5}{5} - \frac{2t^3}{3} \right] + C$$

$\cos x = t$

(4) odd - odd

use = $\sin 3\theta$

$$\int \sin^3 x - \cos^5 x \quad = \int \sin^2 x \cdot \cos^5 x \, (\sin x \, dx)$$

$$\int (1 - \cos^2 x) \cos^5 x \, (\sin x \, dx)$$

$$- \int (1-t^2) t^5 \cdot dt$$

Kya

$-(1-t^2) t^5 \cdot dt$

$- t^5 - t^7 \cdot dt$

$- \frac{t^6}{6} + \frac{t^8}{8} - C$

(5) Even - odd -

Q. $\int \sin^3 x \cdot \cos^3 x \cdot dx$

$$\frac{1}{8} \int 2^3 \sin^3 x \cdot \cos^3 x \, dx$$

$$\frac{1}{8} \int (\sin 2x)^3 \, dx$$

$$\frac{1}{8} \int (\sin 2x) (\sin^2 2x \, dx)$$

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7.1 and 7.2 7.3 Complete

Method 2

$$\sin^3 x \cdot \cos^3 x \cdot dx$$

$$\int \sin^3 x \cdot \cos^2 x (\cos x dx)$$

$$\int \sin^3 x (1 - \sin^2 x)(\cos x dx)$$

$$\int t^3 (1 - t^2)\, dt$$

Q.13 and 15

$$I = \int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha} dx$$

Careful

$$I = \int \frac{(2\cos^2 x - 1) - (2\cos^2 \alpha - 1)}{\cos x - \cos \alpha}$$

(15) $\int \tan^3 2x \cdot \sec 2x\, dx$

$\int (\tan^2 2x) \tan 2x \cdot \sec 2x\, dx$

$\int (\sec^2 2x - 1) \tan 2x \cdot \sec 2x\, dx.$

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Q.14, 17, 20

14, 17, 20

Hint:-

14 $1 + \sin 2n$ Perfect Sq:

$(\cos n + \sin n)^2 = t$

17 $\frac{\sin^3 n}{\sin^2 n \cos^2 n}$.

20 use trig. formula $\cos 2n$ and factorised

21

Formula

Ex:- 7.4 and 7.7

(1) $\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a} + C$

(2) $\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \log\left|x+\sqrt{x^2-a^2}\right| + C$

(3) $\int \frac{1}{\sqrt{a^2+x^2}}\,dx = \log\left|x+\sqrt{a^2+x^2}\right| + C$

x=a²+a

(4) $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}$

(5) $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right| + C$

(6) $\int \sqrt{a^2+x^2}\,dx = \frac{x}{2}\sqrt{a^2+x^2} + \frac{a^2}{2}\log\left|x+\sqrt{a^2+x^2}\right| + C$

(7) $\int \frac{1}{a^2-x^2}\,dx = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + C$

(8) $\int \frac{1}{x^2-a^2}\,dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + C$

(9) $\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$

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4 mark Type-1

(A) $I = \int \frac{1}{\sqrt{ax^2+bx+c}}\,dx$

(B) $I = \int \sqrt{ax^2+bx+c}\,.\,dx$

(C) $I = \int \frac{1}{ax^2+bx+c}$ → Q..E can not be factorise

Ex:1 $I = \int \frac{1}{\sqrt{9x^2+6x+4}}$

$9x^2+6x+4$ ($\frac{2}{3}\times\frac{1}{2}$ always)

$9\left[x^2+\frac{6^2}{9_3}x+\frac{4}{9}\right]+\left(\frac{1}{3}\right)^2-\frac{1}{9}$

$= 9\left[\left(x+\frac{1}{3}\right)^2+\left(\frac{1}{\sqrt{3}}\right)^2\right]$

$I = \int \frac{1}{\sqrt{9\left[\left(x+\frac{1}{3}\right)^2+\left(\frac{1}{\sqrt{3}}\right)^2\right]}}\,dx.$

$= \frac{1}{3}\int \frac{1}{\sqrt{\left(x+\frac{1}{3}\right)^2+\left(\frac{1}{\sqrt{3}}\right)^2}}\,dx.$

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$$= \frac{1}{3}\left[\log\left|\left(x+\frac{1}{3}\right)+\sqrt{\frac{9}{9}\left(x+\frac{1}{3}\right)^2+\left(\frac{1}{\sqrt{3}}\right)^2}\right]\right.$$

$$= \frac{1}{3}\left[\log\left|\frac{3x+1}{3}+\frac{1}{3}\sqrt{9x^2+6x+4}\right.\right.$$

(18) $\int \frac{1}{\sqrt{7-6x-x^2}}$

x^2+6x-7

x^2+6x

$$I = \int \frac{1}{\sqrt{-[x^2+6x-7]}}\,dx.$$

$$I = \int \frac{1}{\sqrt{-(x^2+6x-7+(3)^2-9}}$$

$$I = \int \frac{1}{\sqrt{-[(x+3)^2-(4)^2]}}$$

$$I = \int \frac{1}{\sqrt{(4)^2-(x+3)^2}}$$

$$\left[\because \int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}+c\right]$$

$$I = \sin^{-1}\frac{(x+3)}{4}+c.$$

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6 Mark Type-2

(A) $$I = \int \frac{px+q}{\sqrt{ax^2+bx+c}}$$

(B) $$I = \int (px+q) \cdot \sqrt{ax^2+bx+c}$$

(C) $$I = \int \frac{px+q}{ax^2+bx+c}$$

Can not be factorise

$$\Rightarrow \boxed{\text{Put } px+q = A\frac{d}{dx}(ax^2+bx+c)+B}$$

Q.23 $$I = \int \frac{5x+3}{\sqrt{x^2+4x+10}}\,dx.$$

$$5x+3 = A(2x+4)+B \quad - (1)$$

$$5 = 2A \Rightarrow A = 5/2$$

$$3 = 4A+B \Rightarrow B = -7$$

$$5x+3 = \frac{5}{2}(2x+4) - 7$$

$$I = \int \left[\frac{\frac{5}{2}(2x+4)}{\sqrt{x^2+4x+10}} - \frac{7}{\sqrt{x^2+4x+10}}\right] dx$$

$$I = I_1 + I_2$$

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$$I_1 = \frac{5}{2}\int \frac{2x+4}{\sqrt{x^2+4x+10}}\,dx.$$

Put $x^2+4x+10 = t$

$(2x+4)dx = dt$

$$\frac{5}{2}\int \frac{dt}{\sqrt{t}} = \frac{5}{2}\times 2\sqrt{t}$$

Q. $$I = \int (3x+5)\sqrt{1-4x-x^2}\,.dx.$$

$$\int (3x+5)\sqrt{-(x^2+4x-1)}\,dx.$$

$$3x+5 = A(-2x+4)+B \quad -(1)$$

$$B = -2A \quad \Rightarrow A = -\frac{3}{2}$$

$$5 = 4A+B$$

$$5-6 = B \Rightarrow B = -1$$

Put in eq^n (1)

$$3x+5 = -\frac{3}{2}(-2x+4)-1$$

$$I = \int \left[-\frac{3}{2}(2x+4)-1\right]\sqrt{1-4x-x^2}\,.dx$$

$$I = \int \left[-\frac{3}{2}(-4-2x)-1\right]\times\sqrt{1-4x-x^2}\,.dx$$

$$I = I_1 - I_2 \quad -(2)$$

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$$I_1 = \int \frac{-3}{2}(-4-2x) \cdot \sqrt{(1-4x-x^2)} \cdot dx$$

$$= \frac{-3}{2}\int \sqrt{t}\, dt$$

$$I_2 = \int \sqrt{1-4x-x^2}\, dx$$

Perfect type.

Q. $\int (-3x+5)\sqrt{1+4x+x^2} \cdot dx$

$$-3x+5 = A(4+2x) + B$$

$$-3 = 2A$$

$$A = \frac{-3}{2}$$

$$5 = 4A + B$$

$$5+$$

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Q. $I = \int \frac{x^2}{\sqrt{x^6 + a^6}} = \int \frac{x^2}{\sqrt{(x^3)^2 + (a^3)^2}} \cdot dx$

$= \frac{1}{3} \int \frac{dt}{\sqrt{t^2 + (a^3)^2}}$ $\quad x^3 = t$

$3x^2 \cdot dx = \frac{dt}{3}$

$\frac{1}{3} \log |t + \sqrt{t^2 + a^6}|$ $\quad \int \sqrt{a^2 + a^2} \cdot dx$

Q. $I = \int (x+3) \sqrt{3 - 4x - x^2}$

$x + 3 = A(-4 - 2x) + B$

$1 = -2A \Rightarrow A = \frac{-1}{2}$

$3 = -4A + B$

$3 + 4^2\left(\frac{-1}{2}\right) = B$

$3 - 2 = B \Rightarrow B = 1$

$x + 3 = \frac{-1}{2}(-4 - 2x) + 1$

$I = \int \left[\frac{-1}{2}(-4 - 2x) + 1\right] \sqrt{3 - 4x - x^2}$

$I = I_1 + I_2$

$I_1 = \frac{-1}{2}(-4 - 2x) \sqrt{3 - 4x - x^2}$ $\quad 3 - 4x - x^2 = t$

$-4 - 2x\, dx = dt$

$\Rightarrow -\frac{1}{2} \int \sqrt{t}\, dt$

$I_2 = \int \sqrt{3 - 4x - x^2}$

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EX ⇒ 7.5

Partial fraction!

$$I = \int \frac{P(x)}{Q(x)}\,dx$$

- Proper rational fn → degree of $P(x) < Q(x)$
- Improper rational fn → Dividing method → $P(x) \geq Q(x)$

$$Q(x) \overline{)\,P(x)\,(\,A}$$
$$R$$

$$\left[\frac{P(x)}{Q(x)} = A + \frac{R}{Q(x)}\right]$$

R.F		P.F
(1) $\frac{1}{(x-a)(x-b)}$	=	$\frac{A}{(x-a)} + \frac{B}{(x-b)}$
(2) $\frac{1}{(x-a)(x-b)^2}$	=	$\frac{A}{(x-a)} + \frac{B}{(x-b)} + \frac{C}{(x-b)^2}$
(3) $\frac{1}{(x-a)(bx^2+cx+d)}$ (can not be factor)	=	$\frac{A}{(x-a)} + \frac{Bx+C}{bx^2+cx+d}$

(4) $\frac{(x^2-a^2)(x^2-b^2)}{(x^2+c^2)(x^2+d^2)}$ = Put $x^2 = y$

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IBG

75

Que 8 $I = \int \frac{3x-1}{(x-1)(x-2)(x-3)}\,dx$

Let $\frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x-3)}$ — (1)

$3x-1 = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$

$x = 1 \Rightarrow 2 = A(-1)(-2) \Rightarrow A = 1$

$x = 2 \;;\; 5 = B(1)(-1) \Rightarrow B = -5$

$x = 3 \;;\; 8 = C(2)(1) \Rightarrow C = 4$

$$\int \frac{3x-1}{(x-1)(x-2)(x-3)}\,dx = \int \left[\frac{1}{x-1} - \frac{5}{x-2} + \frac{4}{x-3}\right] dx$$

$$\log(x-1) - 5\log(x-2) + 4\log(x-3) + C$$

Imp.

Q $I = \int \frac{1-x^2}{x(1-2x)}\,dx$

$x - 2x^2 \;\big)\; 1 - x^2 \;\big(\; \frac{1}{2}$

$\frac{x}{2} - x^2$

$1 - \frac{x}{2}$

$-\frac{1}{2}x^2$

$+2x^2$

$\frac{3}{2}x$

(x^2+1)

$-(x^2-1)$

$x - 2x^4$

$(x-1^3)$

$-(2x^2-x)$

$x^2 - 1^2$

$2x^2 - x$

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$$\int \frac{1-x^2}{x(1-2x)}dx = \int\left[\frac{1}{2} + \frac{(1-x/2)}{x(1-2x)}\right]dx$$

$$I = \frac{x}{2} + \left[\int \frac{1-x/2}{x(1-2x)}\,dx + C_1 \right] \quad \text{—(1)}$$

Let $I' = \int \frac{1-x/2}{x(1-2x)}$

let $\frac{1-x/2}{x(1-2x)} = \frac{A}{x} + \frac{B}{1-2x}$ —(2)

$$1 - x/2 = A(1-2x) + Bx.$$

at $x = \frac{1}{2}$ $\quad (1 - \frac{1}{4}) = B/2$

$$\frac{3}{4} = B/2 \Rightarrow B = \frac{3}{2}$$

at $x = 0$ $\quad A = 1$

Put value of A & B in eq (2)

~~Method-2~~

$$I = \int \frac{1-x^2}{x(1-2x)}dx.$$

$$I = \int \frac{(1-2x)+2x}{x(1-2x)}dx - \int \frac{x^2}{x(1-2x)}$$

$$= \int \left(\frac{1}{x} + \frac{2}{1-2x}\right) + \frac{1}{2}\left(\frac{(1-2x)-1}{(1-2x)}\right.$$

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$$\Rightarrow \int \left(\frac{1}{x} + \frac{2}{1-2x}\right) dx + \frac{1}{2}\int 1 - \left(\frac{1}{1-2x}\right)$$

Q.7 $I = \int \frac{x}{(x-1)(x^2+1)}$

$$\frac{x}{(x-1)(x^2+1)} = \frac{A}{(x-1)} + \frac{Bx+C}{x^2+1}$$

$$x = A(x^2+1) + (Bx+C)(x-1)$$

$= A$

$x=1$; $1 = 2A \Rightarrow A = 1/2$

$x=0$ $\quad 0 = A - C \Rightarrow C = A = \frac{1}{2}$

$$0 = A + B \Rightarrow B = -A \Rightarrow -\frac{1}{2}$$

$$I = \int \frac{1/2}{x-1} + \frac{-1/2\,x + 1/2}{x^2+1}\, dx$$

$$\frac{1}{2}\log(x-1) \quad \frac{-1}{2\times 2}\frac{2x}{x^2+1} + \frac{1}{2}\frac{1}{x^2+1}$$

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Important

Ex. $\int \frac{1}{x(x^4+1)}\,dx$.

$\int \frac{1}{x^5(1+\frac{1}{x^4})}\,dx$ Put $1+\frac{1}{x^4}=t$

$-\frac{4}{x^5}\,dx = dt$

$\frac{dx}{x^5} = \frac{dt}{-4}$

$-\frac{1}{4}\int \frac{dt}{t}$

$-\frac{1}{4}\log(t)$

Q. $\int \frac{1}{x(x^n+1)}\,dx$

$\frac{1}{x^{n+1}(1+\frac{1}{x^n})}$

$\frac{1}{-n}\int$

$1+\frac{1}{x^n}=t$

$-\frac{n}{x^{n+1}}\,dx = dt$

$\frac{dx}{x^{n+1}} = \frac{dt}{-n}$

Q. $I = \int \frac{1}{x^4-1}$

$\int \frac{1}{(x^2+1)(x^2-1)} \longrightarrow \frac{1}{2}\int \frac{x^2+1}{(x^2+1)(x^2-1)} - \frac{(x^2-1)}{(x^2+1)(x^2-1)}$

$x^2 = y$

$\int \frac{1}{(y+1)(y-1)}$

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$$\int \frac{1}{(y+1)(y-1)}$$

$$\int \left[\frac{A}{y+1} + \frac{B}{y-1}\right] dx$$

$$A\int \frac{1}{x^2+1}\,dx + B\int \frac{1}{x^2-1}\,dx$$

Q. $\int \frac{1}{x^4+1}\,dx.$

Q. $I = \int \frac{x^2}{x^4+1}\,dx$

Learn

$1 = \frac{1}{2}[(x^2+1) - (x^2-1)]$

$x^2 = \frac{1}{2}[(x^2+1) + (x^2-1)]$

$$I = \frac{1}{2}\int \left[\frac{(x^2+1)}{(x^4+1)} - \frac{(x^2-1)}{(x^4+1)}\right] dx$$

$$I = \frac{1}{2}\int (I_1 - I_2)$$

$$I_1 = \int \frac{x^2+1}{x^4+1}\,dx$$

$$= \int \frac{x^2\left(1+\frac{1}{x^2}\right)}{x^2\left(x^2+\frac{1}{x^2}-2\right)+2}$$

$$= \int \frac{1+1/x^2}{\left(x-\frac{1}{x}\right)^2 + (\sqrt{2})^2}\,dx$$

Put $x - \frac{1}{x} = t$

$$\left(1+\frac{1}{x^2}\right)dx = dt$$

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Q. 9

2008
Q. CBSE

$\int \sqrt{\tan x}\, dx$

Put $\sqrt{\tan x} = t$ $\quad \tan x = t^2$

$\sec^2 x\, dx = 2t\, dt$

$dx = \frac{2t}{1+t^2 \tan^2 x}$

$\int t dx.$

$\int t \times \frac{2t}{1+t^4}$

$\int \frac{2t^2}{1+t^4}$

Q. 2.

$\int \frac{(t^2+1)}{(t^4+1)} + \frac{(t^2-1)}{(t^4+1)}$

$I_1 + I_2$

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$\int \sin^{-1}x \cdot 1\, dx$
I II

Ex: 16

* By Part :

$$\int (u \cdot v)\, dx = u\int v\, dx - \int \left[\frac{dy}{dx} \times \int v\, dx\right] dx.$$
I II

I L A T E
↓ ↓ ↓ ↓ ↘ exp.
Inverse log Algebric trigo

* $\int \sin^{-1}x \cdot 1\, dx$ and $\int \log x \cdot 1\, dx$
I II I II

* $\int \log x \cdot 1\, dx.$
I II

$= (\log x)\, x - \int \left[\frac{1}{x} \times x\right] x$

$= x \cdot \log x - x$

$x(\log x - 1) + c$ Ans.

* $\int (\sin^{-1}x \cdot 1)\, dx$

$= (\sin^{-1}x)\, x - \int \left[\frac{1}{\sqrt{1-x^2}} \times x\right] dx$

$\frac{x}{\sqrt{1-x^2}}$ $1-x^2 = t$
$-2x\,dx = dt$

$= x \sin^{-1}x -$

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PLATE

Q. $\int x \cdot \log x \, dx$.

$\int \log x \cdot x \, dx$

$= (\log x) \frac{x^2}{2} - \int \left[\frac{1}{x} \times \frac{x^2}{2}\right] dx$

$= (\log x) \frac{x^2}{2} - \int \left(\frac{x}{2}\right) dx$

$= (\log x) \frac{x^2}{2} - \frac{1}{2}\left[\frac{x^2}{2}\right] + C$

Q. $\int \underset{II}{x} \cdot \underset{I}{\tan^{-1} x} \, dx$.

PLATE

$\int \tan^{-1} x \cdot x \, dx$

$= (\tan^{-1} x) \frac{x^2}{2} - \int \left(\frac{1}{1+x^2} \times \frac{x^2}{2}\right) dx$

$= -\frac{1}{2} \int \left(\frac{1+x^2}{1+x^2} - \frac{1}{1+x^2}\right)$

$\frac{x^2}{2} \tan^{-1} x - \frac{1}{2}(x - \tan^{-1} x \cdot dx) + C$

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Q.17 – 22 Do all Que

Very Important

* $\int e^x [f(x) + f'(x)] dx = e^x f(x) + C$.

Q.18 $\int e^x \left(\frac{1+\sin x}{1+\cos x}\right) dx$

$\Rightarrow \int e^x \left(\frac{1+2\sin^2 x/2}{2\cos^2 x/2}\right] dx$ divide N and D by $2\cos^2 x/2$

$\int e^x \left(\frac{1}{2}\sec^2 x/2 + \tan x/2\right] dx$

$= \int e^x [f'(x) + f(x)] dx$

$= e^x f(x) + C$

$= e^x \times \tan x/2 + C$

Q.17 $I = \int e^x \left(\frac{x}{(1+x)^2}\right)$

$\int e^x \left(\frac{1}{1+x} - \frac{1}{(1+x)^2}\right) = \frac{e^x}{1+x}$

Q.22 $\int \sin^{-1}\left(\frac{2x}{1+x^2}\right) =$ (2 tan)

$x = \tan\theta$

$2\int (\tan^{-1} x - 1)$.

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7.8 Definate Integration as the Limit of Sum

$$I = \int_a^b f(x) = \lim_{h \to 0} h\{f(a) + f(a+h) + f(a+2h) + \cdots + f(a+(n-1)h)\}$$

$nh = b - a$; $\lim_{h \to 0} \frac{1}{h} = 0$

Q.3

$$I = \int_2^3 x^2 \, dx$$

$nh = 3 - 2 = 1$

$$I = \int_{a=2}^{a=3} x^2 \, dx$$

$\Sigma 1 = n$

$\Sigma n = n(n-1)$

$\Sigma n^2 = \frac{n(n-1)(2n+1)}{6}$

$\Sigma n^3 = \left[\frac{n(n+1)}{2}\right]^2$

$\Sigma n = \frac{a(r^n - 1)}{r-1}$

$$= \lim_{h \to 0} h\{2^2 + (2+h)^2 + (2+2h)^2 + \cdots + (2+(n-1)h)^2$$

$$= \lim_{h \to 0} h\left(2^2 \Sigma 1 + h^2(1^2 + 2^2 + \cdots (n-1)^2) + 4h(1 + 2 + \cdots (n-1))\right)$$

$$= \lim_{h \to 0} h\left[2^2 \Sigma 1 + h^2 \Sigma (n-1)^2 + 4h \Sigma (n-1)\right]$$

$$= \lim_{h \to 0} h\left(4n + h^2 \times \frac{(n-1) \cdot n(2n-1)}{6} + \frac{4h(n-1)n}{2}\right]$$

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7.9 Qo.

$$= \lim_{h\to 0}\left[4(nh) + \frac{(nh)^3\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)}{6} + \frac{4(nh)^2\left(1-\frac{1}{n}\right)}{8}\right]$$

$$= \left(4 + \frac{2}{6} + \frac{4}{2}\right) = 18/3 \text{ Ans}$$

Ex: 7.9.

$$I = \int_a^b f(x)\,dx = [\phi(x)]_a^b = \phi(b) - \phi(a)$$

Ex: 7.10

Substitution method:

Q.2 $$I = \int_0^{\pi/2} \sqrt{\sin\phi}\cdot \cos^5\phi\, d\phi$$

$$\int_0^{\pi/2} \sqrt{\sin\phi}\,\cos^4\phi\,(\cos\phi\, d\phi)$$

$$\int_{\phi=0}^{\phi=\pi/2} \sqrt{\sin\phi}\,(1-\sin^2\phi)^2(\cos\phi\, d\phi)$$

$$\int_0^1 t^{1/2}(1-t^2)^2\,dt$$ | Put $\sin\phi = t$, $\cos\phi\, d\phi = dt$

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Very Imp.

79)

Q. $\int_1^2 e^{2x}\left(\frac{1}{x}-\frac{1}{2x^2}\right)dx$

$2x = t \Rightarrow x = t/2$

$dx = \frac{dt}{2}$

$x = 1 \Rightarrow t = 2$

$x = 2 \Rightarrow t = 4$

$= \int_2^4 e^t\left[\frac{2}{t}-\frac{2}{t^2}\right]$

$= 2\int_2^4 e^t\left(\frac{1}{t}-\frac{1}{t^2}\right)dt$

$f(t) + f'(t)$

$2\left[\frac{e^t}{t}\right]_2^4$

$2\left[\frac{e^4}{4}-\frac{e^2}{2}\right]$ Ans

1.10

Q.3

$\int_0^1 \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

Put $x = \tan t$

$t = \tan^{-1} x$

$x = 0 \quad t = 0$

$x = 1 \quad t = \pi/4$

$I = 2\int_0^{\pi/4} (\tan^{-1}, 1)\,dx$

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Ex- 7.11

Properties of definat integrals :

① $\int_a^b f(x) = \int_a^b f(t)\,dt$

② $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

③ $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

★ (4) $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

★ ⑤ $\int_{-a}^{a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)\,dx & ; f(-x) = f(x) \text{ (Even)} \\ 0 & ; f(-x) = -f(x) \text{ (odd)} \end{cases}$

★ ⑥ $\int_0^{2a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)\,dx & f(2a-x) = f(x) \\ 0 & f(2a-x) = -f(x) \end{cases}$

Ex: P-6 $\int_0^{\pi} \log \sin x\,dx = \int_0^{2\pi/2} \log \sin x\,dx$

$= 2\int_0^{\pi/2} \log \sin x\,dx$

Don't use directly in board exam

(i) $\int_0^{\pi/2} \log \sin x\, dx = -\frac{\pi}{2}\log 2 = \int_0^{\pi/2} \log \cos x\, dx$

(ii) $\int_0^{\pi/2} \log \tan x\, dx = 0 = \int_0^{\pi/2} \log \cot x\, dx$

(iii) $\int_0^{\pi/2} \log \operatorname{cosec} x\, dx = \frac{\pi}{2}\log 2 = \int_0^{\pi/2} \log \sec x\, dx$

(1) 90%.

Prove $= I = \int_0^{\pi/2} \log \sin x\, dx = -\frac{\pi}{2}\log 2$ — (1)

[By prop - $\int_0^a f(x)\, dx = \int_0^a f(a-x)\, dx$]

$I = \int_0^{\pi/2} \log \cos x\, dx$ — (2)

(1) + (2)

$2I = \int_0^{\pi/2} \log\left(2\frac{\sin x \cos x}{2}\right) dx$

$2I = \int_0^{\pi/2} \log \sin 2x\, dx - \int_0^{\pi/2} \log 2\, dx$

$2I = I' - \log 2 \times \frac{\pi}{2}$ — (3)

Now $I' = \int_0^{\pi/2} \log \sin t\, dt$

$2x = t$
$dx = dt/2$
$2x = t$

By Prop 8.

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$f(2a-t) = f(\pi - t) = f(t)$

$I = \frac{1}{2} \times 2 \int_0^{\pi/2} \log \sin t \cdot dt$

Put
$2x = t$
$dx = \frac{dt}{2}$
$x = 0 ; \ t = 0$
$x = \frac{\pi}{2} ; \ t = \pi$

By Prop. P,

$P' = 0 \int_0^{\pi/2} \log \sin x \, dx$

$P' = I$ [put in eq^n ③]

$I = -\log 2 \times \frac{\pi}{2}$ Ans

$\tan \quad \cot x$
$\frac{\sin}{\cos} : \frac{\text{op}}{\text{b}}$

(2) $I = \int_0^{\pi/2} \log \tan x \, dx$ — ①

[By Prop. $= \int_0^a f(x) dx = \int_0^a f(a-x) dx$]

$I = \int_0^{\pi/2} \log \cot x \, dx$ — ②

① + ②

$2I = \int_0^{\pi/2} \log x \cot x \, dx$

$2I = \int_0^{\pi/2} \log 1 \, dx$

$\tan x \cdot \cot x = 1$

$\frac{b}{c} \times \frac{c}{b} = 1$

$\log 1 = 0$

$I = 0$ Ans

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(3) $I = \int_0^{\pi/2} \log \operatorname{Cosec} x \, dx.$

$$I = \int_0^{\pi/2} \log\left(\frac{1}{\sin x}\right) dx$$

$$I = \int_0^{\pi/2} \log(\sin x)^{-1} dx$$

$$I = -\int_0^{\pi/2} \log \sin x \, dx$$

$$I = \log 2 \times \frac{\pi}{2} \text{ Ans}$$

ex 7.11

Q.3

$$I = \int_0^{\pi/2} \frac{\sin^{3/2} x \, dx}{\sin^{3/2} x + \cos^{3/2} x} \quad \text{—(1)}$$

$$I = \int_0^{\pi/2} \frac{\cos^{3/2} x}{\cos^{3/2} x + \sin^{3/2} x} \quad \text{—(2)}$$

(1) + (2)

$$2I = \int_0^{\pi/2} 1 \cdot dx$$

$$2I = \frac{\pi}{2}$$

$$I = \frac{\pi}{4} \text{ Ans}$$

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Q. $I = \int_0^{\pi/4} \log(1+\tan x)\,dx$ — (1)

$$I = \int_0^{\pi/4} \log\left(1+\tan\left(\frac{\pi}{4}-x\right)\right)dx$$

$$= \int_0^{\pi/4} \log\left[1+\frac{1-\tan x}{1+\tan x}\right]dx$$

$$= \int_0^{\pi/4} \log\left[\frac{2}{1+\tan x}\right]dx$$

$$= \int_0^{\pi/4} \log 2 - \log(1+\tan x)\,dx \quad -(2)$$

(1)+(2)

$$2I = \int_0^{\pi/4} \log 2\,dx$$

$$2I = \log 2 \times \frac{\pi}{4}$$

$$I = \log 2 \times \frac{\pi}{8}\,dx$$

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***

0.12 $I = \int_0^{\pi} \frac{x\,dx}{1+\sin x}$ — (1)

$I = \int_0^{\pi} \frac{(\pi - x)\,dx}{1+\sin x}$ — (2)

(1) + (2)

$2I = \int_0^{\pi} \frac{\pi \cdot dx}{1+\sin x}$

$2I = \pi \int_0^{\pi} \frac{1}{1+\sin x} \times \frac{1-\sin x}{1-\sin x}$

$2I = \pi \int_0^{\pi} \frac{1-\sin x}{\cos^2 x}\,dx$

$2I = \pi \int_0^{\pi} \sec^2 x\,dx - \int \sec x \tan x\,dx$

$2I = \pi$

**

16 $I = \int_0^{\pi} \log(1+\cos x)\,dx$ — (1)

By

$I = \int_0^{\pi} \log(1-\cos x)\,dx$ — (2)

$2I = \int_0^{\pi} 2 \log \sin x\,dx$

$I = \frac{2\pi}{2} \int_0^{\pi} \log \sin x\,dx$

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$= 2\int_0^{\pi/2} -\sin x \, dx$ — Proof is required

$= 2 \times \frac{\pi}{2} \log 2$

$= \pi \log 2$

(6) $\int_2^8 |x-5| \, dx$



$-\int_2^5 (x-5) + \int_5^8 (x-5) \, dx$

Miscel

Q.33 $\int_1^4 |x-1| + |x-2| + |x-3| \, dx$

$\int_1^4 |x-1| dx + \int_1^4 |x-2| \, dx + \int_1^4 |x-3| \, dx$

$\int_1^4 (x-1) \, dx + \int_1^4 (x-2) \, dx - \int_1^4 (x-3) \, dx$

$\left[\frac{x^2}{2} - x\right]_1^4 + \left[\frac{x^2}{2} - 2x\right]_1^4 - \left[\frac{x^2}{2} - 3x\right]_1^4$

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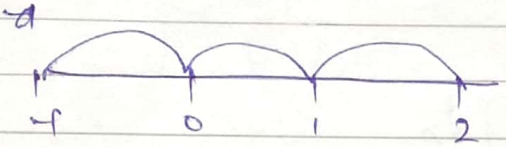
$$\left(\frac{16}{2}-4\right)-\left(\frac{1}{2}-1\right)+\left(\frac{16}{2}-8\right)-\left(\frac{1}{2}-2\right)-\left(\frac{16}{2}+2\right)-\left(\frac{1}{2}-3\right)$$

$$4+\frac{1}{2}-\frac{3}{4}+4-\frac{5}{2}$$

Q. $\int_{-1}^{2} |x^3-x|\,dx$

$$\int_{-1}^{2} |x(x+1)(x-1)|\,dx$$





$$\int_{-1}^{0} - \int_{0}^{1} + \int_{1}^{2}$$

Q. $\int_{-1}^{3/2} |x \sin \pi x|\,dx$ (Cos πx)

$x \sin \pi x = 0$

$x = 0 \; ; \; \sin \pi x = 0$

$x = 0 \; ; \; \sin \pi x = 0$

$\pi x = n\pi$

$x = n$

$n = 1$

if $\sin\theta = 0$

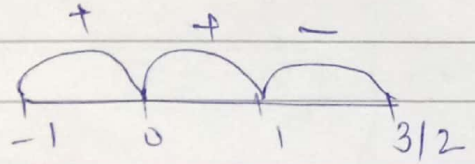
$\theta = n\pi$

$\cos\theta = 0$

$\theta = (2n+1)\pi/2$

$\tan\theta = 0$

$\theta = n\pi$



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3,4,5

$$\rightarrow \int_{-1}^{+1} (x \sin \pi x)\, dx - \int_{1}^{3/2} (x \sin \pi x)\, dx$$

* Miscellaneous Ex.

Q.4

$$I = \int \frac{1}{x^2 (x^4+1)^{3/4}}\, dx.$$

$$\int \frac{1}{x^5 \underbrace{\left(1+\frac{1}{x^4}\right)^{3/4}}_{t}}\, dx$$

$$1 + \frac{1}{x^4} = t$$

$$\int \left(t^{-3/4}\right) dt.$$

Q.7 $I = \int \frac{\sin x}{\sin(x-a)}$ ① $x - a = l$

$$= \int \frac{\sin(\overbrace{x-a}^{A} + \overbrace{a}^{B})}{\sin(x-a)}$$

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(18) $$I = \int \frac{1}{\sqrt{\sin^3 x \sin(x+\alpha)}}\,dx$$

$$I = \int \frac{1}{\sqrt{\sin^3 x (\sin x \cos\alpha + \cos x \sin\alpha)}}\,dx = \int \frac{1}{\sqrt{(\sin^4 x)(\cos\alpha + \cot x \cdot \sin\alpha)}}$$

$$I = \int \frac{\operatorname{cosec}^2 x}{\sqrt{\underbrace{\cos\alpha}_{\text{Const.}} + \cot x \cdot \sin\alpha}}\,dx$$

$$\sqrt{\cos\alpha + \cot x \cdot \sin\alpha} = t$$

$$\therefore 2\sqrt{t}$$

(19) $$I = \int \frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}}$$

$$I = \frac{2}{\pi}\int \left[\sin^{-1}\sqrt{x} - \left(\frac{\pi}{2} - \sin^{-1}\sqrt{x}\right)\right] dx$$

$$= \frac{2}{\pi}\int \left(2\sin^{-1}\sqrt{x} - \pi/2\right) dx$$

$\cos t = \sqrt{1-\sin^2 t}$

$$= \frac{4}{\pi}\int \sin^{-1}\sqrt{x}\,dx - \int 1\,dx$$

Put
$\sin^{-1}\sqrt{x} = t$

$\sqrt{x} = \sin t$

$x = \sin^2 t$

$dx = \sin 2t\,dt$

$$\frac{4}{\pi}\int (t \cdot \sin 2t)\,dt$$

(I) (II)

definite try

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Maths Paper
21 March 2018

20 $I = \int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\, dx$

$I = \int \left(\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} \times \frac{1-\sqrt{x}}{1-\sqrt{x}} \right)$

$\frac{1-\cos t}{\sin t}$

$I = \int \sqrt{\frac{(1-\sqrt{x})^2}{1^2-(\sqrt{x})^2}}$

$I = \int \sqrt{\frac{(1-\sqrt{x})^2}{1^2-(\sqrt{x})^2}}$

Imp.

$I = \int \frac{1-\sqrt{x}}{\sqrt{1-x}}\, dx$

Put $\sqrt{x} = \cos t$

$x = \cos^2 t$

$dx = -2\cos t \sin t\, dt$

$2 \sin 2t$

$-2\int (\cos t - \cos^2 t)\, dt$

Imp
Q. 26

$= \int_{\pi/6}^{\pi/3} \frac{\sin x + \cos x}{\sqrt{\sin 2x}}\, dx$

$\cos^4 x$ higher

$I = \int_0^{\pi/4} \frac{\sin x \cos x}{\cos^4 x + \sin^4 x}$

$= \int_0^{\pi/4} \frac{\sin x \cdot \cos x / \cos^4 x}{1 + \tan^4 x}$

$\tan^2 x = t$

$= \int_0^{\pi/4} \frac{\tan x \cdot \sec 2x}{1 + (\tan^2 x)^2}$

$2 \tan x \sec^2 x\, dx = dt$

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2,3o, 20-41
Very Imp

Q. 21

$$I = \int_0^{\pi/2} \frac{\cos^2 x}{\cos^2 x + 4\sin^2 x}\, dx$$

$$I = \int_0^{\pi/2} \frac{1}{1 + 4\tan^2 x}\, dx.$$

Partial fraction

$$I = \int_0^{\pi/2} \frac{1}{1 + (2\tan x)^2}$$

$$I = \int_0^{\pi/2} \frac{1}{1+4\tan^2 x} \times \frac{\sec^2 x}{(1+\tan^2 x)}\, dx$$

$\tan x = t$
$\sec^2 x\, dx = dt$

$$= \int_0^{\infty} \frac{dt}{(1+4t^2)(1+t^2)} \qquad t^2 = y$$

$$= \frac{1}{(1+4y)(1+y)} = \frac{A}{1+4y} + \frac{B}{1+y}$$

$$A\int \frac{1}{1+(2t)^2} + B\int \frac{1}{1+t^2}$$

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$\int \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}$

Q. 28 $\int_{\pi/6}^{\pi/3} \frac{\sin x + \cos x}{\sqrt{\sin 2x}}\, dx.$

Ex-41 $\int \sqrt{\tan x} + \sqrt{\cot x}\, dx$

$\int_{\pi/6}^{\pi/3} \frac{\sin x + \cos x}{\sqrt{1 - \sin^2 x - 1}}\, dx$

$\sqrt{2} \int \frac{\sin x + \cos x}{\sqrt{2 \sin \cos x}}$

$\int \frac{\sin x + \cos x}{\sqrt{2 \sin x}}$

$\int_{\pi/6}^{\pi/3} \frac{\sin x + \cos x}{\sqrt{1 - (\sin x - \cos x)^2}}$

$(\sin x - \cos x)^2 = t$ $\quad d(\sin x - \cos x) = dt$

Q. 30

$= \int_0^{\pi/4} \frac{\sin x + \cos x}{9 + [16 - [1 + \sin 2x - 1]}$

$= \int_0^{\pi/4} \frac{\sin x + \cos x}{25 - 16(\sin x - \cos x)^2}$

$\sin x - \cos x = t$

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$$\int e^x [f(x) + f'(x)] dx = e^x f(x)$$

V. good

EX-40

$$I = \int \left[\log(\log x) + \frac{1}{(\log x)^2} \right] dx$$

$\log x = t$

$x = e^t$

$dx = e^t dt$

$$I = \int e^t \left(\log t + \frac{1}{t^2} \right) dt$$

$$\int e^t \left[\left(\log t + \frac{1}{t} \right) - \left(\frac{1}{t} - \frac{1}{t^2} \right) \right] dt$$

$$\int e^t \underbrace{\left(\log t + \frac{1}{t} \right)}_{f(x)} dt + \int e^t \underbrace{\left(\frac{1}{t} - \frac{1}{t^2} \right)}_{f'(x)} dt$$

$$= e^t (\log t) - \frac{e^t}{t} + c$$

$$e^t \left(\log t - \frac{1}{t} \right) c.$$

$$x \left[\log(\log x) - \frac{1}{\log x} \right] + c$$

D.

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8.32 $\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx$

$\int_0^{\pi} \frac{(\pi - x) \tan x}{\sec(\pi - x) + \tan(\pi - x)} dx$

$\pi \int_0^{\pi} \frac{-\tan x}{\sec x + \tan x} - \frac{x \tan x}{\sec(\pi - x) + \tan(\pi - x)}$

$I = \pi \int_0^{\pi} \frac{\tan x}{[\sec x + \tan x} - \frac{x \tan x}{\sec x + \tan x}$

$2I = \pi \int_0^{\pi} \frac{-\tan x}{(\sec x + \tan x)} dx$

$2I = \pi \int$

Ans $I = \int_0^{\pi} \frac{(\pi - x) \tan x}{[\sec x + \tan x]} dx$

$I = \int \frac{\pi \tan x - x \tan x}{[\sec x + \tan x]} dx$ — (2)

$2I = \pi \int_0^{\pi} \frac{\tan x}{\sec x + \tan x} dx$ rationalisation

~~divide & multiply sec x - tan~~

$2I = \pi \int_0^{\pi} \tan x (\sec x + \tan x)$ $\therefore 2I = \pi \int_0^{\pi} \tan x \sec x + \underbrace{\tan^2 x}_{(\sec^2 x - 1)} dx$

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Ex: 44 $I = \int_0^{\pi} \frac{x}{a^2\cos^2 x + b^2\sin^2 x}\,dx$

$= \int_0^{\pi} \frac{x \sec^2 x}{a^2 + b^2\tan^2 x}\,dx$

$2I = \pi \int_0^{2\cdot\pi/2} \frac{\sec^2 x\,dx}{a^2 + b^2\tan^2 x}$ Put $\tan x = t$

P6

$2I = 2\pi \int_0^{\infty} \frac{dt}{a^2 + (bt)^2}$ $\tan x = t$

$\sec^2 x\,dx = dt$

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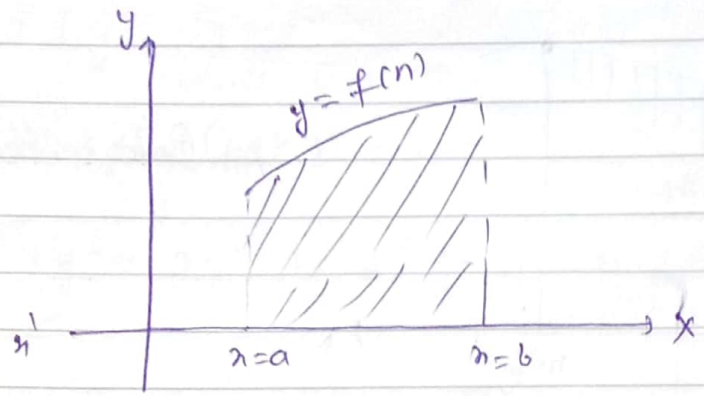
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1 Que
6 Marks

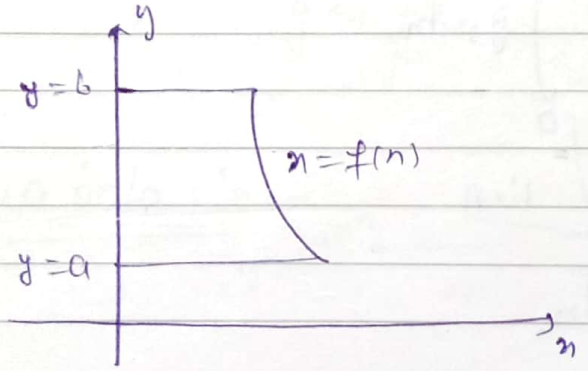
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Area under the curve

* Area under the curve along the x-axis and y-axis



$$\text{Required Area} = \int_a^b y\,dx = \int_a^b f(x)\,dx = \text{finite value (Square unit)}$$



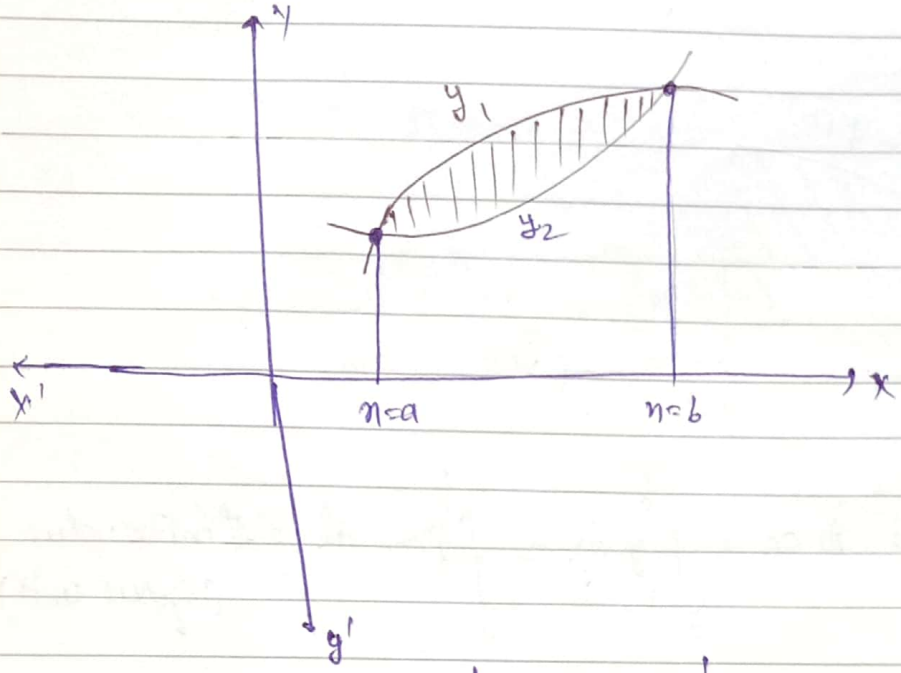
$$\text{Required Area} = \int_a^b x\,dy$$

$$= \int_a^b f(y)\,dy$$

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two

* Area bounded by curves :



All Intersection point

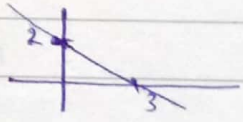
$$\text{Required Area} = \int_a^b y_1\,dx - \int_a^b y_2\,dx$$

T.A − U.A

$$= \int_a^b (y_1 - y_2)\,dx.$$

* Standard graph :

(1) Line : $ax + by = c$ $\quad 4x + 6y = 12$ (3, 2)



(2) $y = mx + c$

(3) $\frac{x}{a} + \frac{y}{b} = 1$

(4) $x\cos\alpha + y\sin\alpha = p$

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(2) $x = \sqrt{7}y$

(iii) Circle :

(1) $x^2 + y^2 = a^2$

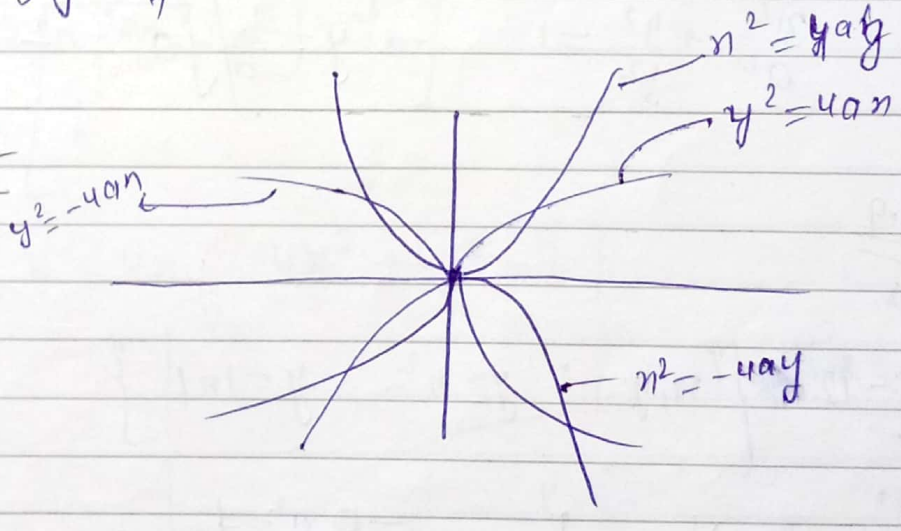
(2) $(x-h)^2 + (y-k)^2 = a^2$

(3) $x^2 + y^2 + 2gx + 2fy + c = 0$

$$c = \begin{cases} g = -\left(\frac{\text{Coffi'cent of } x}{2}\right) \\ f = -\left(\frac{\text{Coffi'cent of } y}{2}\right) \end{cases}$$

$$r = \sqrt{g^2 + f^2 - c}$$

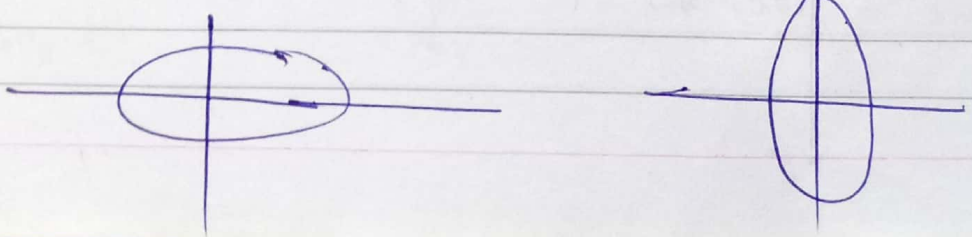
(3) Parabola :



(4) ellipse :

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad (a > b)$

$a < b$



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(1) $I = \int x^n dx = \frac{x^{n+1}}{n+1} + C$

$$I = \int \frac{1}{\sqrt{x}} dx = 2\sqrt{x} + C$$

$$I = \int \frac{1}{x^2} = -\frac{1}{x}$$

$$I = \int \frac{1}{x^3} = -\frac{1}{2x^2}$$

(2) $$\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2} \times \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} x/a$$

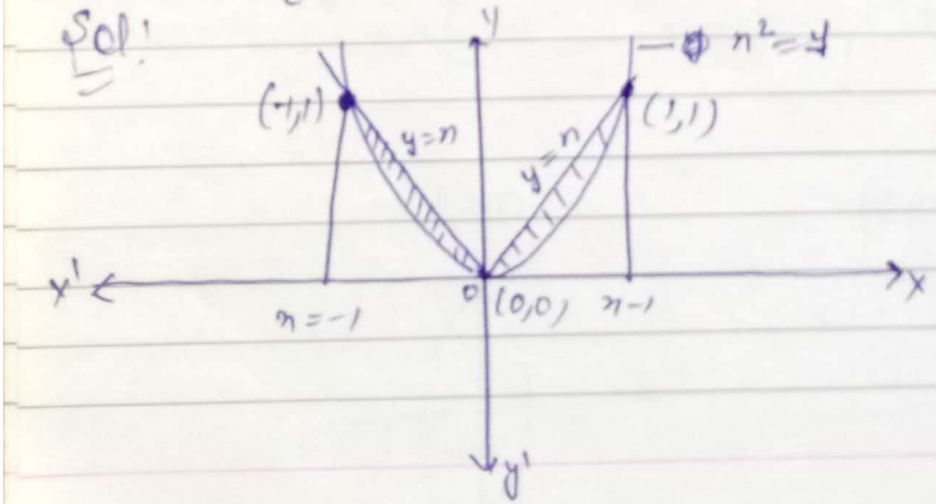
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \Rightarrow y = \frac{b}{a}\sqrt{a^2 - x^2}$$

Exmp. 10.9

Ex-12

Misc-12 $\{(x, y) : y \geq x^2, y = |x|\}$

Sol:



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Intersection point

$x^2 = y$ $\quad y = x$ $\quad y = -x$

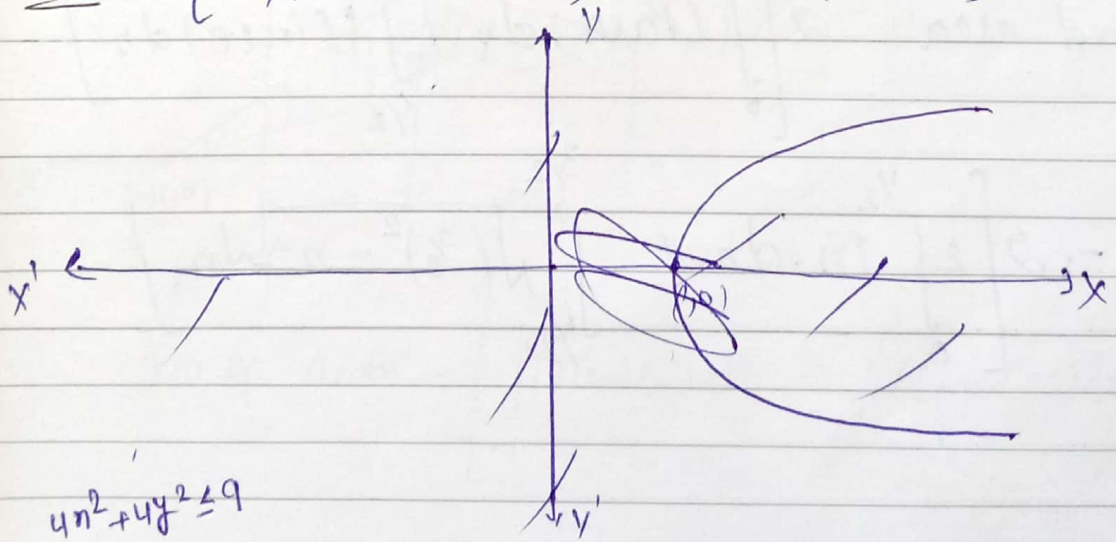
$x^2 = x$	$x^2 = -x$
$x^2 - x = 0$	$x^2 + x = 0$
$x(x-1) = 0$	$x(x+1) = 0$
$x = 0 \; ; \; x = 1$	$x = 0 \quad x = -1$
$y = 0 \; , \; x = 1$	$y = 0 \quad y = 1$
$(0,0) \quad (1,1)$	$(0,0) \quad (-1,1)$

Required Area $= 2\left[\int_0^1 (\text{line})\,dx - \int_0^1 (\text{Parab.})\,dx\right]$

$= 2\left[\int_0^1 (x - x^2)\,dx\right]$

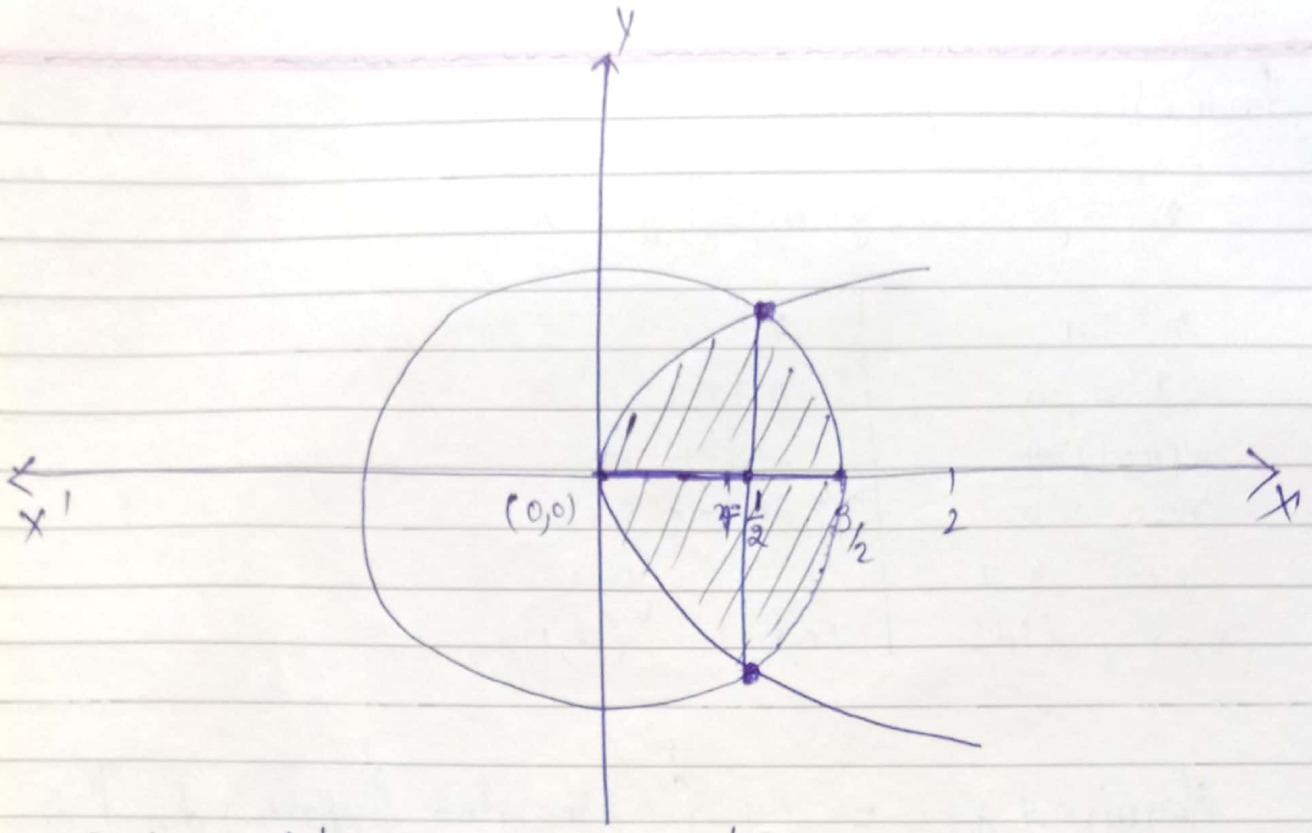
Ans:

Q.15 $\{(x,y) : y^2 \le 4x, \quad 4x^2 + 4y^2 \le 9\}$



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$y^2 = 4x$ and $x^2 + y^2 = \left(\frac{3}{2}\right)^2$



Intersection Point of given graph

$y^2 = 4x$ $\qquad x^2 + y^2 = \left(\frac{3}{2}\right)^2 = 9/4$

$4x^2 + 4y^2 = 9$

$$\text{Required area} = 2\left[\int_0^{1/2} (\text{Para})\,dx + \int_{1/2}^{3/2} (\text{Circle})\,dx\right]$$

$$= 2\left[2\int_0^{1/2} \sqrt{x}\,dx + \int_{1/2}^{3/2} \sqrt{(3/2)^2 - x^2}\,dx\right]$$

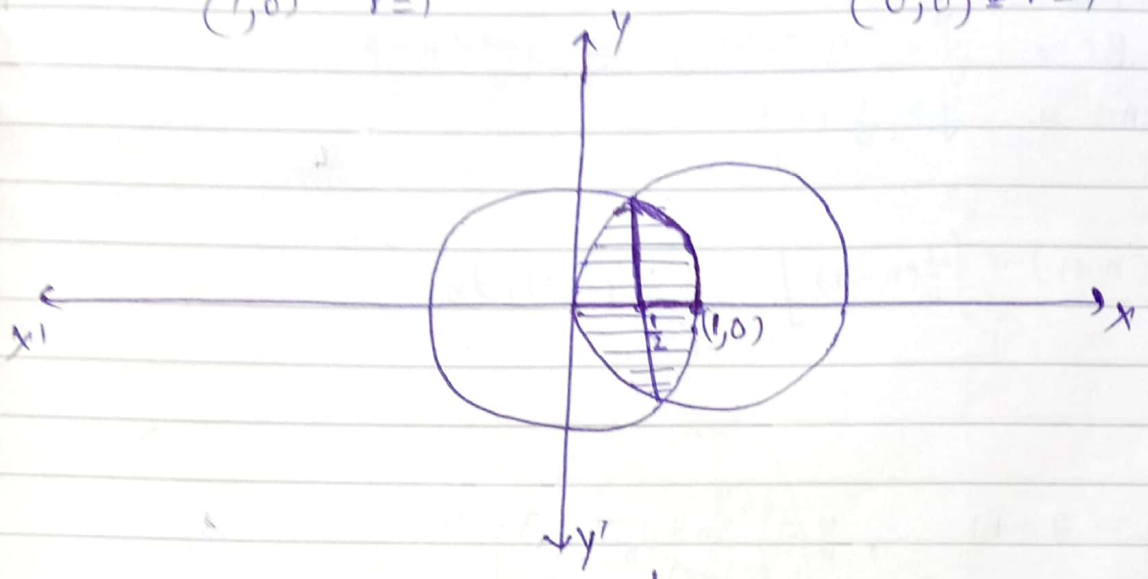
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Ques

(8.2) (2)

$C_1 = (x-1)^2 + y^2 = 1$ and $C_2 = x^2 + y^2 = 1$

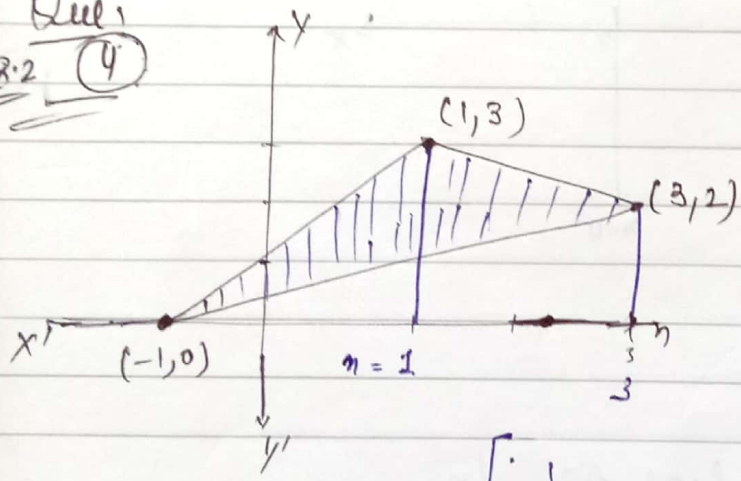
$(1,0)$ $r=1$ $(0,0)$ $r=1$



$$\text{Req. Area} = 4\int_{1/2}^{1} \sqrt{1-x^2}\, dx.$$

Ques:

8.2 (4)



$$\text{Area of } \Delta ABC = \left[\int_{-1}^{1} (\text{line } AB)\, dx + \int_{1}^{3} (\text{line } BC)\, dx\right] - \left[\int_{-1}^{3} (\text{line } AC)\, dx\right]$$

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$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Line AB $\Rightarrow$ $y = 3/2\,(x+1)$

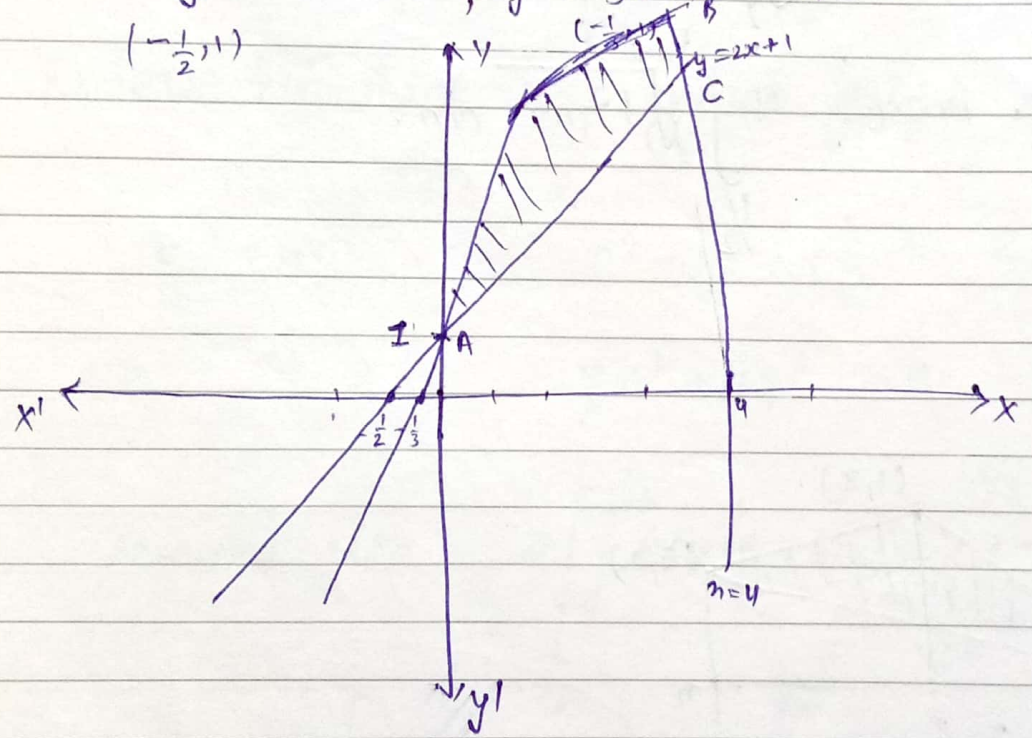
Line BC $\Rightarrow$ $y = -1/2\,(x-7)$

Line AC $\Rightarrow$ $y = \frac{1}{2}(x+1)$

$$\left[\frac{3}{2}\int_{-1}^{1}(x+1) + \int_{1}^{3}\frac{-1}{2}(x-7)\right] - \int_{-1}^{3}\frac{1}{2}(x+1)\,dx.$$

Ques-5

$y = 2x+1$, $y = 3x+1$, $x = 4$



$$\Delta ABC = \int_{0}^{4}(\text{line } AB - \text{line } AC)\,dx$$

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Que:

Example-1 $x^2+y^2 = 8x$ $y^2 = 4x$.

$x^2 - 8x + y^2 = 0$ $g = (-\frac{8}{2}) = 4$

$f = 0$

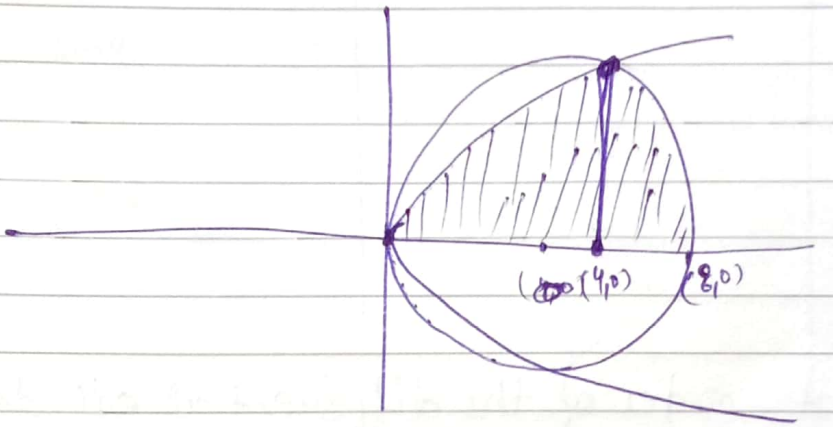
$r = \sqrt{g^2+f^2-c^2} = 4$

$(4,0)$

$y^2 \geq 4x$

↓

Outside the parabola



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19/12/17 Chapter - 9 10 Mark

Differential equation

(9.1)

D.E	order	Degree.

* Order: Heighest order of the differential eq^{n} decode order of D

Degree: Power of Heighest order decode degree of Differential eq^{n}.

Note: If given D.E is not a polynomial eq^{n} in derivative the degree is not define

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Q.1

D.E	order	Degree
(1) $\left(\frac{dy}{dx}\right)^2 + \left(\frac{d^2y}{dx^2}\right)^1 + y = 1$	2	1
2) $(y'')^2 + y' = 1$	2	2
(3) $\frac{d^2y}{dx^2} + \sin y' = 4$	2	N.D
4) $\frac{d^2y}{dx^2} + e^{dy/dx} = 4$	2	Not defiw

Q.2

Types Of Solution:

(1) General Solution: No. of arbitary Constant is equal to order of D

(2) Perticular Sol^n: No. of arbitary Constant is always equal to zero

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M-3 $y = ae^{3x} + be^{-2x}$

$ye^{2x} = ae^{5x} + b$

$2y e^{2x} + e^{2x} \cdot y' = 5ae^{5x} + 0$

Q5

$y = e^x (a\cos x + b \sin x)$. — (1)

$y = e^x a\cos x + e^x b \sin x$

$y' = e^x a\cos x - e^x a \sin x + e^x b \sin x + e^x b \cos x$.

$y' = ae^x \cos x - ae^x \sin x + be^x \sin x + be^x \cos x$

$y' = e^x (-a\sin x + b\cos x) + y$ — (2)

$\Rightarrow$

$y'' = -e^x (a\cos x + b\sin x) + y' - y + y'$

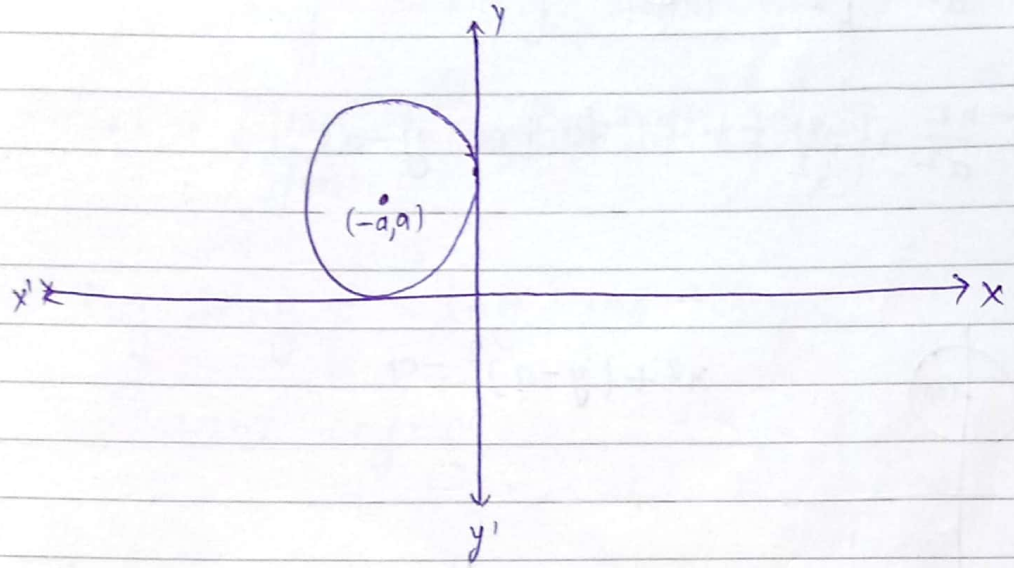
$y'' = -y + y' - y + y'$

Q. write the DE of Circle $(x-h)^2 + (y-k)^2 = r^2$ where h and k are arbitrary const.

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Ex - 9.3

formation of D.E :



$$(x+a)^2 + (y-a)^2 = a^2 \quad \text{---(1)}$$

$$x^2 + a^2 + 2ax + y^2 + a^2 - 2ay = a^2$$

Diff.

$$2x + 0 + 2a + 2yy' - 2ay' = 0$$

$$a(1-y') = \frac{-(x+xyy')}{(1-y')}$$

$$a = \frac{x+yy'}{y'-1}$$ put this eq^n in eq^n (1).

Q. 9.3

(2) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad (a<b)$ ——(1)

Diff

$$\frac{2x}{a^2} + \frac{2yy'}{b^2} = 0 \quad \text{——(2)}$$

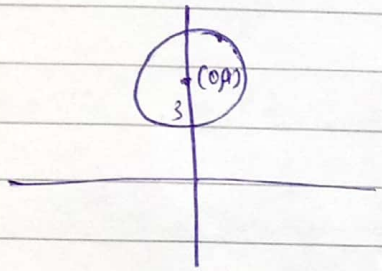
Diff $= \frac{1}{a^2} + \frac{1}{b^2}\left[yy'' + (y')^2\right] = 0$ —— (3)

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Hyperbola :- $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ [foci on x-axis]

$-\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (foci on y-axis

Q.10



$x^2 + (y-a)^2 = a$

* first Method of for solving D.E ① $\frac{dy}{dx}$

1) Variable Seprati'on Method:- (9.4)

$$f(x) \cdot dy - f(y) \cdot dx = 0$$

$$f(x) \cdot \frac{dy}{dx} - f(y) \frac{dx}{dx}$$

9.4
Q.4 $\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$

$$\sec^2 x \tan y + \sec^2 y \tan x \frac{dy}{dx} = 0$$

$$\sec^2 y \tan x \frac{dy}{dx} = -\sec^2 x \tan y$$

$$\frac{dy}{dx} = \frac{-\sec^2 x \tan y}{\sec^2 y \tan x}$$

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$$\int \frac{\sec^2 y}{\tan y}\,dy = -\int \frac{\sec^2 x}{\tan x}\,dx$$

$$\because \left[\int \frac{f'(x)}{f(x)}\,dx = \log|f(x)| + c\right]$$

$$\log \tan y = -\log \tan x + \log c$$

$$\tan x \tan y = c$$

Que (11)

$$(x^3+x^2+x+1)\frac{dy}{dx} = 2x^2+x \quad ; \quad y=, \; x=0$$

$$dy = \frac{2x^2+x}{x^2(x+1)+(x+1)}\,dx$$

$$\int dy = \int \frac{2x^2+x}{(x+1)(x^2+1)}$$

Q.18

slope of tangent $= 2\left(\right.$ (x, y) — (−4, −3)

(−2, 1)

x y

$$\frac{dy}{dx} = 2\left(\frac{3+y}{4+x}\right)$$

$$\int \frac{dy}{y+3} = 2\int \frac{dx}{x+y}$$

$x=2$, $y=1$

$c = ?$

$$\log(y+3) = 2\log(x+4) + c \quad \text{— (1)}$$

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(19) Given: $\frac{dV}{dt} = a$

at $t=0$; $r=3$

after $t=3$; $r=6$

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

from eq ①

$$4\pi r^2 \frac{dr}{dt} = a \qquad 4\pi \int r^2 dr = a \int dt$$

$$4\pi \int_3^6 r^2 dr = a \int_0^1 1 dt.$$

$$4\pi \left[\frac{r^3}{3}\right]_3^6 = a \left[t\right]_0^1$$

$$4\pi (72 - 9) = a$$

$$a = 84\pi$$

$$4\pi \int r^2 dr = a \int dt \Rightarrow 4\pi \int_3^r r^2 dr = 84\pi \int_0^t dt$$

$$4\pi \left(\frac{r^3}{3}\right)_3^r = 84\pi (t)_0^t$$

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$$= \frac{4\pi}{3}\left[r^3 - 27\right) = 84\pi \times t$$

$$r^3 - 27 = 63t$$

$$r = (63t + 27)^{1/3}$$

Q.22

Given $\frac{dN}{dt} \propto N$

$t = 0$; $N = 100000$
$t = 2$; $N = 1,10,000$
$t = ?$; $N = 200\,000$

Let N be the total no. of bacteria in a culture at Present

$$\frac{dN}{dt} = kN$$

$$\int_{100000}^{110000} \frac{dN}{N} = k\int_0^2 dt$$

$$\frac{1}{2}\log\frac{11}{10} = 2k$$

$$\int_{1\,Lack}^{2\,Lack} \frac{dN}{N} = \frac{1}{2}\log\frac{11}{10}\int_0^t dt$$

$$\log 2 = \frac{1}{2}\log\frac{11}{10} \times t$$

$$t = \frac{\log 2}{\frac{1}{2}\log\frac{11}{10}}$$

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6 March

9.5

* Homogeneous Diff. equation :

(1) $\frac{dy}{dx} = f\left(\frac{y}{x}\right)$ | (2) $\frac{dx}{dy} = f\left(\frac{x}{y}\right)$

$\frac{dy}{dx} = \frac{x^2+y^2}{x^2+(xy)}$

Homogeneity Power Same

Q. $\frac{dy}{dx} = \frac{x^2+y^2}{x^2+(xy)}$

Q. Let $f(x,y) = \frac{x^2+y^2}{x^2+xy}$

$f(\lambda x, \lambda y) = \frac{\lambda^2(x^2+y^2)}{\lambda^2(x^2+xy)}$

$f(\lambda x, \lambda y) = \lambda^0 \cdot f(x,y)$

Method

put $y = vx$

$\frac{dy}{dx} = v \cdot 1 + x \cdot \frac{dv}{dx}$

$v + x\frac{dv}{dx} = \frac{x^2(1+v^2)}{x^2(1+v)}$

variable sep.

$x\frac{dv}{dx} = \frac{1+v^2}{1+v} - \frac{v}{1}$

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$$x\frac{dv}{dx} = \frac{1+v^2-v-v^2}{1+v} = \frac{1-v}{1+v}$$

$$\int \frac{1+v}{1-v}\,dv = \int \frac{dx}{x}$$

$$\int \frac{v+1}{v-1} + 1 + 1 \,.dv = -\int \frac{dx}{x}$$

$$\int \left[\frac{v-1}{v-1} + \frac{2}{v-1}\right] dv = -\int \frac{dx}{x}$$

$$v + 2\log(v-1) = -\log x + C$$

Put

Q.10 $$\left(1+e^{\frac{x}{y}}\right)dx + e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)dy = 0$$

$$\left(1+e^{x/y}\right) + e^{x/y}\left(1-\frac{x}{y}\right)\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{1+e^{x/y}}{e^{x/y}\left(1-\frac{x}{y}\right)}$$

$$\frac{dx}{dy} = -\frac{e^{x/y}\left(1-x/y\right)}{1+e^{x/y}}$$

Put $x = vy$

$$\left\{\frac{dx}{dy} = v\cdot 1 + y\frac{dv}{dy}\right\}$$

$\frac{dx}{dy} = d\left(\frac{x}{y}\right)$

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$$v + y\frac{dv}{dy} = \frac{-e^v(1-v)}{1+e^v}$$

$$y\frac{dv}{dx} = \frac{-e^v(1-v)}{1+e^v} - 1$$

$$y\frac{dv}{dy} = \frac{-e^v + ve^v - v - ve^v}{1+e^v}$$

$$\frac{y\,dv}{dy} =$$

$$\int \frac{1+e^v}{v+e^v}\,dv = -\int \frac{dy}{y} \qquad \int \frac{f'(x)}{f(x)}\,dx = \log|f(x)| + c$$

$$\log(v+e^v).dv = -\log y + \log c$$

Put $V = x/y$ [

Ex 9.6

Linear Diff. Equation :

$$\frac{dy}{dx} + Py = Q$$

where P and Q are f^n of x or Constant

Integral factor $= I.F \Rightarrow e^{\int P\,dx}$.

Sol^n of given diff. eq^n is $y \times (I.F) = \int (Q \times I.F)\,dx$

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(11) $\frac{dx}{dy} + Px = Q$

where P and Q are f^n of y or constant

$I.F = e^{\int P.dy}$

Sol^n: $x \times (I.F) = \int (Q \times I.F)\, dy$

Q.10 $(x+y)\frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{x+y}$

$\frac{dx}{dy} = x+y$

$\frac{dx}{dy} - x = y$

$P = -1 \quad Q = y$

$I.F = e^{\int P.dy} \Rightarrow e^{\int -1\, dy} = e^{-y}$

Sol^n: $x e^{-y} = \int \underset{I}{y} \cdot \underset{D}{e^{-y}}.dy$

Using Integration by Part

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Ex

Q.9 $x\frac{dy}{dx} + y - x + xy\cot x = 0$

divided by x.

$$\frac{dy}{dx} + \frac{y}{x} - 1 + y\cot x = 0$$

$$\frac{dy}{dx} + \left(\frac{1}{x} + \cot x\right)y = 1$$

$$\frac{dy}{dx} + Py = Q$$

$P = \frac{1}{x} + \cot x$ and $Q = 1$

$$I.f = e^{\int P\,dx}$$

$$= e^{\int \left(\frac{1}{x} + \cot x\right)dx}$$

$$= e^{\log x + \log \sin x}$$

$$= e^{\log(x \sin x)}$$

$$I.f = x \sin x.$$

Now sol^n given D.E is

$$y \times (I.f) = \int (Q \times I.f)\,dx$$

$$xy\sin x = \int \underset{II}{1} \times \underset{I}{(x \sin x)}\,dx$$

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Q.8 $(1+x^2)dy + 2xy\,dx = \cot x\,dx$

$$(1+x^2)\frac{dy}{dx} + 2xy = \cot x$$

$$\frac{dy}{dx} + (1+x^2)2xy = \cot x$$

$$\frac{dy}{dx} + (2x + 2x^3)y = \cot x$$

$$I.F = e^{\int P\,dx} \qquad P = 2x + 2x^3 \qquad Q = \cot x$$

$$= e^{\int (2x+2x^3)dx}$$

$$= e^{\left(x^2 + \frac{x^4}{2}\right)}$$

$$(1+x^2)\frac{dy}{dx} + 2xy = \cot x$$

$$\frac{dy}{dx} + \left(\frac{2x}{1+x^2}\right)y = \frac{\cot x}{1+x^2}$$

$$I.F = e^{\int \frac{2x}{1+x^2}} \qquad x^2 = t, \quad 2x\,dx = dt$$

$$= e^{\int \frac{dt}{1+(t)^2}} = e^{\tan^{-1} t}$$

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D.E

Q.4 $(x^3 - 3xy^2)dx = (y^3 - 3x^2y)dy$

Sol^4 $(x^2 - y^2) = C(x^2 + y^2)^2$

$$\frac{dy}{dx} = \frac{x^3 - 3xy^2}{y^3 - 3x^2y}$$

Power same Homogenious

$$\left\{ y = vx, \quad \frac{dy}{dx} = v \cdot 1 + x\frac{dv}{dx} \right\}$$

$$v + x\frac{dv}{dx} = \frac{x^3 - 3v^2x^3}{v^3x^3 - 3vx^3}$$

$$x\frac{dv}{dx} = \frac{1 - 3v^2}{v^3 - 3v} - \frac{v}{1}$$

$$x\frac{dv}{dx} = \frac{1 - 3v^2 - v^4 + 3v^2}{v^3 - 3v}$$

$$\int \frac{v^3 - 3v}{1 - v^4}\, dv = \int \frac{dx}{x}$$

$$-\frac{1}{4}\int \frac{-4v^3}{1 - v^4}\, dv - 3\int \frac{v}{1 - v^4}\, dx = \int \frac{dx}{x}$$

$$-\frac{1}{4}\log(1 - v^4) - 3\int \frac{v}{1 - (v^2)^2}\, dx = \log x + C$$

Put $v^2 = t$

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Q.7 Sol^{n}: $(x+y+1) = A(1-x-y-2xy)$ Ans

Q.E $\quad \frac{dy}{dx} + \frac{y^2+y+1}{x^2+x+1} = 0$

$$\frac{dy}{dx} = -\frac{(y^2+y+1)}{(x^2+x+1)}$$

$$\int \frac{dy}{y^2+y+1} = -\int \frac{dx}{x^2+x+1}$$

$$\int \frac{dy}{(y+\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2} = -\int \frac{dx}{(x+\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2}$$

$$\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2y+1}{\sqrt{3}}\right) = -\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+C$$

$$\frac{2}{\sqrt{3}}\left[\tan^{-1}\left(\frac{2x+2y+2\sqrt{3}}{1-\left(\frac{2y+1}{\sqrt{3}}\right)\left(\frac{2x+1}{\sqrt{3}}\right)}\right)\right] = \frac{C\sqrt{3}}{2}$$

$$\frac{\frac{2}{\sqrt{3}}(x+y+1)}{\frac{2}{3}(1-x-y-2xy)} = \frac{1}{\sqrt{3}}\ln\left(\frac{\sqrt{3}}{2}C\right) = A$$

$$(x+y+1) = A(1-x-y-2xy).$$

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Q.12 $\left[\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right]\frac{dx}{dy} = 1$

$\left[\frac{e^{-2\sqrt{x}} - y}{\sqrt{x}}\right]\frac{dx}{dy} = 1$

$\frac{dx}{dy} = \frac{\sqrt{x}}{e^{-2\sqrt{x}} - y}$

$\frac{dy}{dx} = \left[\frac{e^{-2\sqrt{x}} - y}{\sqrt{x}}\right]$

$\frac{dy}{dx} = \frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}$

$\frac{dy}{dx} + \left(\frac{1}{\sqrt{x}}\right)y = \frac{e^{-2\sqrt{x}}}{\sqrt{x}}$ $\quad \frac{dy}{dx} + Py + Q$

$P = \frac{1}{\sqrt{x}}$ and $Q = \frac{e^{-2\sqrt{x}}}{\sqrt{x}}$ $\quad I.f = e^{\int P.dx}$

$I.f = e^{\int \frac{1}{\sqrt{x}} dx}$

$I.f = e^{2\sqrt{x}}$

Sol^n: $y \times I.f = \int (Q \cdot If)\, dx$

$y \times e^{2\sqrt{x}} = \int \left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} \times e^{2\sqrt{x}}\right)$

$y \times e^{2x} = 2\sqrt{x} + C.$

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Q.11 $(x-y)(dx+dy) = dx-dy$

$$(x-y)\left(1+\frac{dy}{dx}\right) = 1-\frac{dy}{dx}$$

$$(x-y) + (x-y)\frac{dy}{dx} = 1-\frac{dy}{dx}$$

$$\frac{dy}{dx}\left[1+(x-y)\right] = 1-(x-y)$$

$$\frac{dy}{dx} = \frac{1-(x-y)}{1+(x-y)}$$

Put $x-y=t$

$$1-\frac{dy}{dx} = dt \qquad -\frac{dy}{dx} = \frac{dt}{dx} - 1$$

$$\frac{dy}{dx} = 1-\frac{dt}{dx}$$

$$1-\frac{dt}{dx} = \frac{1-t}{1+t}$$

use variable method.

$$\frac{dt}{dx} = 1-\left(\frac{1-t}{1+t}\right)$$

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2015

Q.. $(x-h)^2 + (y-k)^2 = r^2$ — (1)

Diff:

$2(x-h) + 2(y-k)\frac{dy}{dx} = 0$

$(x-h) + (y-k)\frac{dy}{dx} = 0$ — (2)

Diff (2)

$1 + (y-k)\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$

$(y-k)\frac{d^2y}{dx^2} = -\left(\left(\frac{dy}{dx}\right)^2 + 1\right)$

$(y-k) = -\frac{\left(\left(\frac{dy}{dx}\right)^2 + 1\right)}{\left(\frac{d^2y}{dx^2}\right)}$ Put in eq (2)

$(x-h) = +\frac{\left[\left(\frac{dy}{dx}\right)^2 + 1\right]}{\left(\frac{d^2y}{dx^2}\right)} \times \frac{dy}{dx}$ Put in eq (1)

$\frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^2 \left(\frac{dy}{dx}\right)^2}{\left(\frac{d^2y}{dx^2}\right)^2} + \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^2}{\left(\frac{d^2y}{dx^2}\right)^2} = r^2$

(1) $y-k$ in (2)

$x-h$ — eq (1)

Put in eq (1)

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$$\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right] \times \left(1+\left(\frac{dy}{dx}\right)^2\right)}{\left(\frac{d^2y}{dx^2}\right)^2} = r^2$$

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2/12/17

Chapter - 6

A.O.D.

4+6=10 Marks

Ex 6.1

Rate of change :-

* if $y = f(x)$ then $\frac{dy}{dx}$ represent rate of change in y w.r.t x.

	Total Surface Area (cm^2)	Volume (cm^3)
i) Cube	$6 \times (side)^2$	$a^3 = (side)^3$
ii) Cuboid	$2(lb + bh + hl)$	$l \times b \times h$
iii) Cylinder	$2\pi rh + \pi r^2$	$\pi r^2 h$
iv) Cone	$\pi r^2 + \pi r l$	$\frac{1}{3}\pi r^2 h$
v) Sphere	$4\pi r^2$	$\frac{4}{3}\pi r^3$

20 =

Q.2 $\frac{dv}{dx}$ $\frac{ds}{dt}$ $\frac{dx}{dt}$

$\frac{cm^3}{cm}$, $\frac{cm^2}{sec}$, $\frac{cm}{sec}$

Sol : Given $\frac{dv}{dt} = 8\ cm^3/sec.$ Cube

to find $\frac{ds}{dt}$ at $x = 12cm.$

$\because \quad v = x^3$

$\frac{dv}{dt} = 3x^2 \frac{dx}{dt} \quad = \quad \frac{dm}{dt} = \frac{8}{3x^2}$

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Imp. Qu:- 2, 5, 7, 9, 10, 11, 13, 14, 15, 16

Very important Questions

5

11-3/

$s = 6x^2$

$\frac{ds}{dt} = 12x \frac{dx}{dt}$

from eq^n (1)

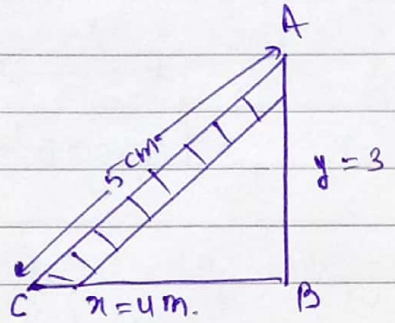
$\frac{ds}{dt} = 12 \times 4 \times 8$

$3n2$

$B^2 = h^2 - p^2$

$\left(\frac{ds}{dt}\right)_{x=12} = \frac{4 \times 8}{12_3} = \frac{8}{3}$ $\frac{cm^2}{sec.}$ Ans.

Q.10



$\frac{dx}{dt} = 2cm/sec$

$\frac{dy}{dt} = ?$

In ΔABC

$x^2 + y^2 = 25$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$x\frac{dx}{dt} + y\frac{dy}{dt} = 0$

$4 \times 2 + 3\frac{dy}{dt} = 0$

$\frac{dy}{dt} = -\frac{8}{3}$

In ΔABC

$y = \sqrt{25 - x^2}$

$y = \sqrt{25 - 16} = \sqrt{9} = 3m.$

$400 \times 2 + 300\frac{dy}{dt} = 0$

$\frac{dy}{dt} = -\frac{8}{3}$ Ans

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Q13

sphere.

$D = \frac{3}{2}(2x+1)$ $\quad r = \frac{3}{4}(2x+1)$

$\frac{dv}{dx} = ?$

$V = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi\left(\frac{3}{4}(2x+1)\right)^3$

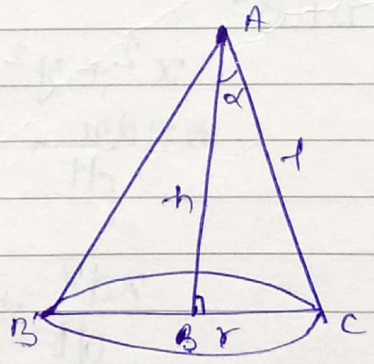
$= \frac{9}{16}\pi(2x+1)^3$

$\frac{dv}{dt} = \ldots$

Q14

$\frac{dv}{dt} = 12 cm^3/sec$

$\frac{dh}{dt} = ?$ $\quad h = 4cm$



$h = \frac{r}{6}$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{\pi}{3}(6h)^2 h$

$V = \frac{36}{3}\pi h^3$

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Note: Miscellaneous 259 PNo.

Example-43

Q 15, 16 — 1 Mark

$$\frac{dv}{dt} = 12\pi \times 3h^2 \cdot \frac{dh}{dt} = 12$$

$$\frac{dh}{dt} = \frac{1}{\pi 3h^2}$$

$$\frac{dh}{dt} = \frac{1}{\pi 3(4)^2} = \frac{1}{3\pi(4)^2} \text{ cm/sec}$$ Ans

Ex-6.3

Tangent & Normal :

* eq^n of tangent : $y - y_1 = \left(\frac{dy}{dx}\right)_{P(x_1,y_1)} (x - x_1)$

where $\frac{dy}{dx}$ represent slope of tangent at a point (x_1, y_1)

* eq^n of Normal : $y - y_1 = -\frac{1}{\left(\frac{dy}{dx}\right)_{P(x_1,y_1)}} (x - x_1)$

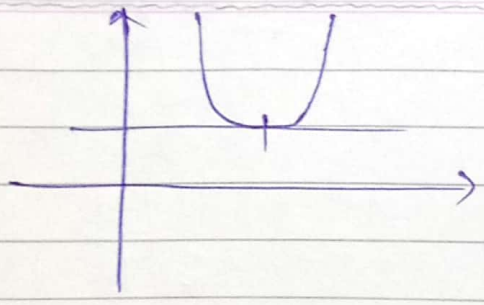
where $-\frac{1}{\frac{dy}{dx}}$ represent slope of Normal.

$$m_1 \times m_2 = -1$$

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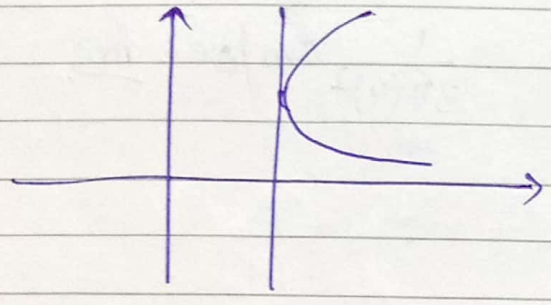
Imp. Qu: 10, 11, 12, 18, 21, 23, 24, 25

(3)



$\left(\frac{dy}{dx}\right)_p = 0$

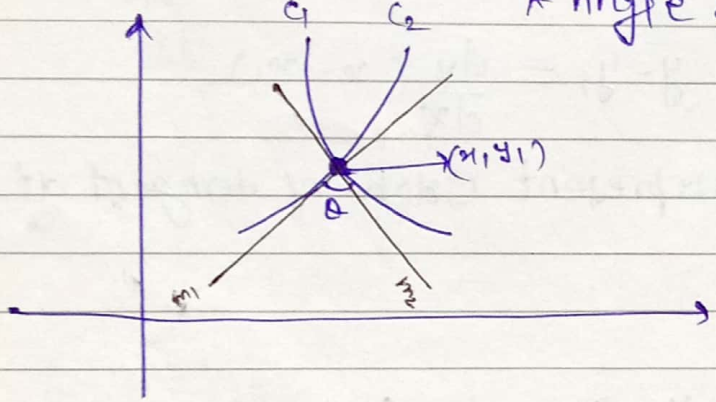
(4)



$\frac{dy}{dx} = \infty = \frac{1}{0}$

$\frac{dx}{dy} = 0$

(5) * Angle b/w two curve.



$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

Q.6

Ans All lines Identical ~~is to~~ ~~to~~ with ~~the~~ tangent

Slope of tangent = Slope of line

$$\frac{dy}{dx} = -1$$

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Given eqⁿ : $y = \frac{1}{x-1}$ —(I) $\quad \frac{dy}{dx} = -\frac{1}{(x-1)^2}$ —(II)

$\frac{dy}{dx} = -1$

from eqⁿ (I) and (II)

$\frac{-1}{(x-1)^2} = +1 \qquad (x-1)^2 = 1$

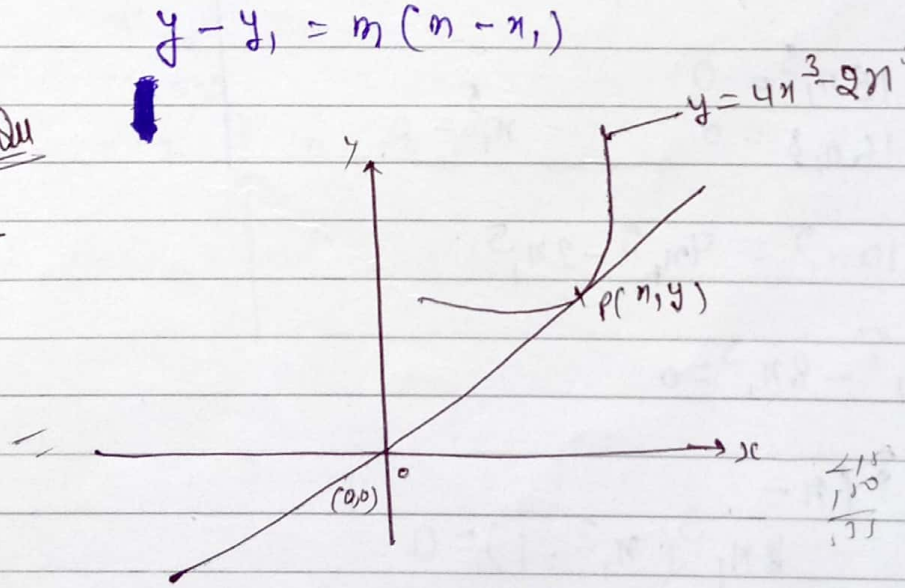
$x-1 = \pm 1$

$x-1 = 1$	$x-1 = -1$
$x = 2$	$x = 0$
$y = 1$	$y = -1$

eqⁿ of a line at a point (2,1)
" " " " " " (0,-1)

$y - y_1 = m(x - x_1)$

Very Imp Qu
Q.18



Slope of line of = Slope of tangent

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$$\frac{y_2 - y_1}{x_2 - x_1} = \left(\frac{dy}{dx}\right)_{P(x_1, y_1)}$$

$$\frac{y_1}{x_1} = 12x_1^2 - 10x_1^4$$

$$y_1 = 12x_1^3 - 10x_1^5 \quad \text{---(1)}$$

Point (x_1, y_1) lie on the curve $y = 4x^3 - 2x^5$

$$y_1 = 4x_1^3 - 2x_1^5 \quad \text{---(2)}$$

From eqn (1) and (2)

$$y_1 = 12x_1^3 - 10x_1^5$$
$$y_1 = 4x_1^3 - 2x_1^5 \quad 3 \times 3$$

$$0 = 0 - 10x_1^5 - 6x_1^5$$

$$-16x_1^5 = 0$$

$$16x_1^5 = 0 \quad \Rightarrow x_1^5 = 0$$

$$12x_1^3 - 10x_1^5 = 4x_1^3 - 2x_1^5$$

$$8x_1^5 - 8x_1^3 = 0$$

$$8x_1^3(x_1^2 - 1) = 0$$

$$8x_1^3(x_1 + 1)(x_1 - 1) = 0$$

$x_1 = 0, 1, -1$ Put in eqn (1)

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same

Q.24 Normal = ?

curve $y = x^3 + 2x + 6$

line $x + 14y + 4 = 0$ — slope $\left(\frac{\text{coeff. of } x}{\text{coeff. of } y}\right)$

Q.25 Same

Q. Normal = ⊥ to line.

Normal ∥ line ↑

$$-\left(\frac{1}{3x^2+2}\right) = -\left(\frac{1}{14}\right)$$

$$\text{Normal} = y - y_1 = \frac{-1}{dy/dx}(x - x_1)$$

$3x^2 + 2 = 14$

$3x^2 = 12$

$x^2 = 4$

$x = \pm 2$

$x = 2$ | $x = -2$

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Ⓑ Max/min 6 mark

Ex 6.2
Ex 6.3
Ex 6.1
Ex 6.4

$\frac{1}{2\sqrt{n}} \times \frac{k}{n^2}$

Viimp.

Q.23

$8k^2 = 1$

Method.

Intersection point of curves $(k^{2/3}$

$C_1 \Rightarrow x = y^2$

$C_2 \Rightarrow xy = k$

$y = \sqrt{n}$

$\frac{dy}{dn} = \frac{1}{2\sqrt{n}}$

$y = \frac{k}{n}$

$-\frac{k}{n^2}$

$n = y^2$

$n = \frac{k}{y}$ $\quad 1 - \frac{1}{3}$

$\frac{2}{3}$

$1 = \frac{y^3}{k}$

$k = y^3$

$y = k^{1/3}$

I.P $(k^{2/3}, k^{1/3})$

$m_1 = \left(\frac{dy}{dn}\right)_P$

$m_2 = \left(\frac{dy}{dn}\right)_P$

$m_1 \times m_2 = -1$

$8k^2 = -1$

Ex-6.2 Increasing or Decreasing f^n :

(1) f^n $f(n)$ be Strictly Increasing iff $f'(n) > 0$ $\forall n \in (a, b)$

(2) " " " Decreasing iff $f'(n) < 0$ $\forall n \in (a, b)$

" " " only increasing iff $f'(n) \geq 0$ $\forall n \in [a, b]$

" " " only Decreasing iff $f'(n) \leq 0$ $\forall n \in [a, b]$

Q.6 (e)

Let $f(n) = (n+1)^3 (n-3)^3$

$f'(n) = (n+1)^3 \cdot 3(n-3)^2 + (n-3)^3 \cdot 3(n+1)^2$

$f'(n) = 3(n-3)^2 (n+1)^2 [(n+1) + (n-3)]$

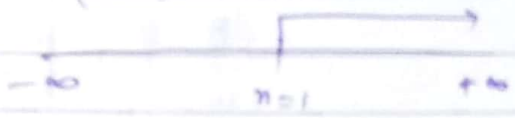
$f'(n) = 6(n-3)^2 (n+1)^2 (n-1)$

For strictly incring $\Rightarrow f'(n) > 0$

$6(n-3)^2 (n+1)^2 (n-1) > 0$

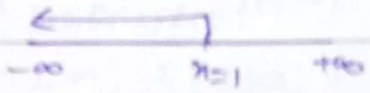
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$(x-1) > 0$



$(1, \infty) - \{3\}$

(2) for S.D



$(-\infty, 1) - \{1\}$

Q.7

$f(x) = \log(1+x) - \frac{2x}{2+x}$ $\quad x > -1$

$f'(x) = \frac{1}{1+x} -$

$y = \log(1+x) - \frac{2x}{2+x}, \quad x > -1$

$\frac{dy}{dx} = \frac{1}{1+x} - \left[\frac{(2+x)\cdot 2 - 2x \cdot 1}{(2+x)^2}\right.$

$\frac{dy}{dx} = \frac{1}{1+x} - \frac{4}{(2+x)^2}$

$= \frac{4 + x^2 + 4x - 4}{(1+x)(2+x)^2} = \frac{4x}{(1+x)(2+x)^2}$

$\frac{dy}{dx} = \frac{x^2}{(1+x)(2+x)^2}$

for $f(x)$ $\quad f'(x) \geq 0$

$+x \geq 0$ But $x \neq 0$

$1+x > 0$

$x > -1$ H.P.d

Book Prob Sum

Q.19 $y = \frac{4\sin\theta}{(2+\cos\theta)} \quad -\theta$

$\frac{dy}{d\theta} = \frac{(2+\cos\theta)(4\cos\theta) + 4\sin^2\theta}{(2+\cos\theta)^2}$ $+\frac{1}{1}$

$\frac{dy}{d\theta} = \frac{8\cos\theta + 4 - (4 + \cos^2\theta + 4\cos\theta)}{(2+\cos\theta)^2}$

$\frac{dy}{d\theta} = \frac{4\cos\theta - \cos^2\theta}{(2+\cos\theta)^2}$

$\frac{dy}{d\theta} = \frac{\cos\theta(4-\cos\theta)}{(2+\cos\theta)^2}$

$\cos\theta \geq 0$ if $\theta \in [0, \frac{\pi}{2}]$

$4 - \cos\theta > 0$ if $\theta \in [0, \frac{\pi}{2}]$

$(2+\cos\theta)^2 > 0$ if $\theta \in [0, \frac{\pi}{2}]$

$\therefore \frac{dy}{d\theta} \geq 0$ if $\theta \in [0, \frac{\pi}{2}]$

Hence f^n is increasing in $[0, \frac{\pi}{2}]$

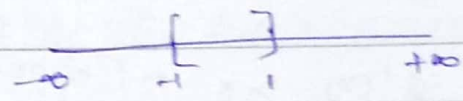
$\frac{dy}{d\theta} = -4\sin\theta(2\sin\theta) - 4\cos\theta(2+\cos\theta)$

$= \frac{dy}{d\theta} = 4(\sin^2\theta - 2\cos\theta + \cos^2\theta)$

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Q.15

$f(x) = x + \frac{1}{x}$

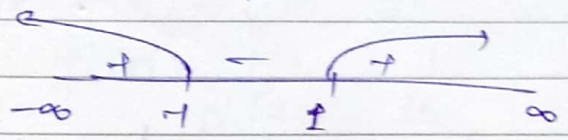
$SI = [-1, 1] \Rightarrow$ 

$f(x) = x + \frac{1}{x}$ $= (-\infty, -1) \cup (1, \infty)$

$f'(x) = 1 - \frac{1}{x^2} = \frac{(x+1)(x-1)}{x^2}$

$f'(x) > 0$

$\frac{(x+1)(x-1)}{x^2} > 0$



$SI = (-\infty, -1) \cup (1, \infty)$

Example-12

$f(x) = 2\sin 3x \qquad x \in [0, \frac{\pi}{2}]$

$f'(x) = 2\cos 3x = 0$

$\cos 3x = 0$

$3x = (2n+1)\frac{\pi}{2}$

$x = (2n+1)\frac{\pi}{6}$

$n=0 \quad ; \quad x = \pi/6$

$n=1 \quad ; \quad x = \frac{\pi}{2}$

$\cos\theta = 0$

$\theta = (2n+1)\frac{\pi}{2}$

$2\sin\theta = 0$

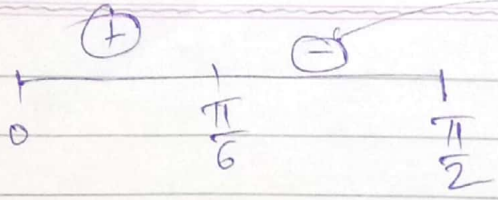
$\theta = n\pi$

$\tan\theta = 0$

$\theta = n\pi$

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sign equation



S. inc. $(0, \frac{\pi}{6})$

only inc. $[0, \frac{\pi}{6})$

~~S. dec~~ $(\frac{\pi}{6}, \frac{\pi}{2})$

only dec $(\frac{\pi}{6}, \frac{\pi}{2}]$

S.I $\Rightarrow f'(x) > 0 \Rightarrow [0, \frac{\pi}{6})$

only Inc. $\Rightarrow f'(x) \geq 0 \quad (\frac{\pi}{6}, \frac{\pi}{2})$

I $\Rightarrow f'(x) \geq 0 \quad [0, \pi/6]$

D $\Rightarrow f'(x) \leq 0 \quad [\frac{\pi}{6}, \frac{\pi}{2}]$

Ex-13

$f(x) = \sin x + \cos x \qquad 0 \leq x \leq 2\pi = x \in [0, 2\pi]$

$f'(x) = \cos x - \sin x.$

$\cos x - \sin x = 0$

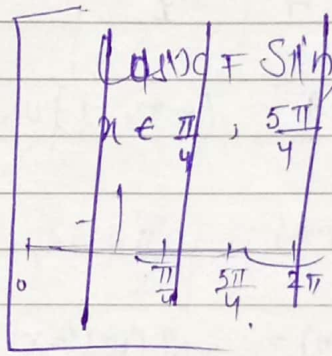
$\tan x - \sin x = 0$

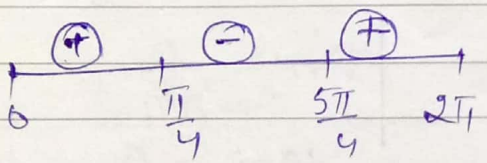
$\tan x = 1 = \tan \frac{\pi}{4}$

$x = n\pi + \pi/4$

$x = 0 \; ; \quad x = \frac{\pi}{4}$

$x = 1 \; ; \quad x =$





I = $(0, \frac{\pi}{4} \cup \frac{5\pi}{4}, 2\pi)$

D = $\frac{\pi}{4}, \frac{5\pi}{4}.$

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Approximation

* Approximations and errors:

If $I = f(x)$
then approximate value of the function is denoted by $f(x) + f'(x) \cdot \Delta x$

(i) $$\boxed{f(x + \Delta x) = f(x) + f'(x) \cdot \Delta x}$$

Q: Find the
(i) $\sqrt{25.3}$

$25.3 = 25 + 0.3$
$x \quad \Delta x$

Let $f(x) = \sqrt{x}$ $\quad x = 5$

$f'(x) = \frac{1}{2\sqrt{x}} = \frac{1}{10}$

$f(x+\Delta x) = f(x) + f'(x) \cdot \Delta x$
$= 5 + \frac{1}{10} \times 0.3$

(2) $.64 \times 4 = 256$;

Q. $(.009)^{1/3} = x^{1/3} \Rightarrow 0.008 + .001$

$(225)^{1/4} = 256 - 1$

$(225)^{1/4} = (256 + 1)^{1/4}$

$x \quad \Delta x \quad : f(x+\Delta x) = f(x) + f'(x)\Delta x.$

$4 + (1)^{1/4}$.

$4 + 8 = 12$

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Ex-6.4 (i) $(0.009)^{1/3}$

$$f(x+\Delta x) = f(x) + f'(x)\cdot\Delta x \quad \text{***}$$

x $\quad$ Δx

$= (0.009)^{1/3} = 0.008 + 0.001$

let $f(x) = x^{1/3}$ $\quad\therefore f'(x) = \frac{1}{3}x^{-2/3}$

(ii) $(0.9999)^{1/4} = 256 - 1$

$= (4)^{4\times\frac{1}{4}}$

* Approximate change

If $y = f(x)$ then app. change in y w.r.t x denoted by $\boxed{\frac{\Delta y}{\Delta x} = \frac{dy}{dx}}$

$$\Delta y = \frac{dy}{dx} \times \Delta x.$$

* Percentage change in a x

$$x = \frac{\Delta x}{x} \times 100$$

Q.4 $\Delta V = ?$ $\quad$ % Change in side $x = 1\%$.

$$\frac{\Delta V}{\Delta x} = \frac{dv}{dx} \qquad \therefore V = x^3$$

$$\frac{dv}{dx} = 3x^2$$

$$\Delta V = 3x^2 \times \frac{x}{100} \quad = \quad \Delta V = 0.03x^3 \ \frac{\Delta y}{m^3}$$

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Q.6 Sphere

- $r = 7\,cm$

  $\Delta r = 0.02\,m$

  $\Delta V = ?$

$$\frac{\Delta V}{\Delta r} = \frac{dv}{dr} \qquad V = \frac{4}{3}\pi r^3$$

- $\Delta V = \frac{dv}{dr} \times \Delta r \qquad \frac{dv}{dr} = 4\pi r^2$

$$\Delta V = 4\pi r^2 \times \Delta r$$

$$= 4\pi \times (7)^2 \times 002\ m^3$$

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Ex- 6.5

Maxima & Minima (1 Que - 6 Mark)

- Local
  - (1) first order derivative
  - (2) second order derivative
- Absolute

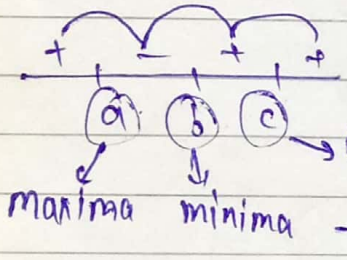
$\frac{dy}{dx} = (x-a)(x-b)(x-c) = 0$

$x = a, b, c.$

$\frac{dy}{dx} = (x-a)(x-b)(x-c)$

~~(1)~~

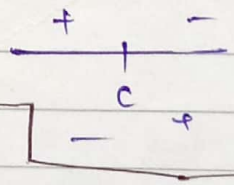
$x = a, b, c$



$\left(\frac{d^2y}{dx^2}\right)_{x=a} < 0 \Rightarrow x = a$ maxima.

$\left(\frac{d^2y}{dx^2}\right)_{x=b} > 0 \Rightarrow x = b$ minima

$\left(\frac{d^2y}{dx^2}\right)_{x=c} = 0 \Rightarrow x = c$ we don't



Absolute:

$f(x)$ [a, b]

$f'(x) = 0$

$x = c_1, c_2, c_3$

Point	$f(x)$
a	$f(a)$
c_1	$f(c_1)$
c_2	$f(c_2)$
c_3	$f(c_3)$
b	$f(b)$

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$\frac{d}{dx}(x^n) = nx^{n-1}$

6.5

Q.3 (v)

$f(x) = x^3 - 6x^2 + 9x + 15$

for maxima/minima $f'(x) = 0$

$f'(x) = 3x^2 - 12x + 9 \Rightarrow$

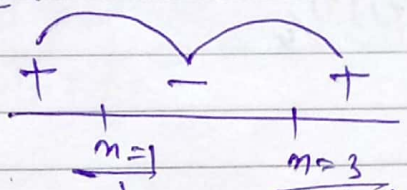
$= 3x^2 - 3x - 9x + 9 = 0$

$= 3x(x-1) - 9(x-1) = 0$

$= (3x-9)(x-1) = 0$

$x = 3, \quad x = 1$

M-1 f.o.D.T:



maxima minima

M-2

Second D.D.T

$f''(x) = 6x - 12$

$f''(1) = 6 - 12 = -6 < 0$

$x = 1$ maxima

$f''(3) = 18 - 12 = 6 > 0$

$x = 3$ minima.

show

$\left(\frac{dy}{dx}\right)_{x<1} =$

$\left(\frac{dy}{dx}\right)_{x>1} =$

$\left(\frac{dy}{dx}\right)_{x>3} =$

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Ques: 8

$f(x) = \sin 2x \qquad [0, 2\pi]$

$f'(x) = 2\cos 2x$

for max/min $f'(x) = 0$

$2\cos 2x = 0 \qquad 2x = (2n+1)\frac{\pi}{2}$

$\cos 2x = 0$

$2x = (2n+1)\pi/2$

$x = (2n+1)\pi/4$

$n = 0, \quad x = \frac{\pi}{4}$

$n = 1 \quad x = \frac{3\pi}{4}$

$n = 2 \quad ; \quad x = \frac{5\pi}{4}$

$n = 3 \quad ; \quad x = \frac{7\pi}{4}$

Points	$f(x) = \sin 2x$	
0	0	
$\pi/4$	1	absolute max.
$3\pi/4$	-1	
$5\pi/4$	1	
$7\pi/4$	-1	
2π	0	

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1.36 to 16

13 to 16 Q.

Q13 Let two Nos. are x and y.

Given $x+y=24$ — (1)

(max) $P = xy$

$P = x(24-x)$

$P = 24x - x^2$

$\frac{dP}{dx} = 24x - x^2 = 0$

$\frac{dP}{dx} = 24 - 2x$

$\frac{d^2P}{dx^2} = -2 < 0$

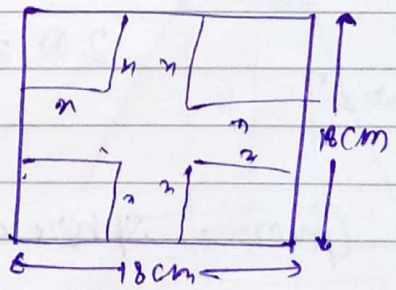
$\left(\frac{d^2P}{dx^2}\right) = -2 < 0$

$x = 12$, maxima

$y = 12$

Q. 17 | 18

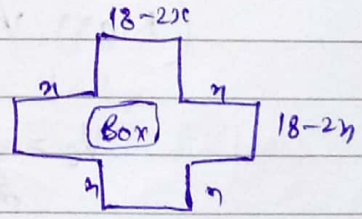
Sol:



Cube = 18cm.

$V = l \times b \times h$

(max) $V = (18-2x)^2 \times x$



$\frac{dV}{dx} = (18-2x)^2 \cdot 1 + x \cdot 2(18-2x)(-2)$

$\frac{dV}{dx} = (18-2x)(18-6x) = 0$

$x = 9 \quad x = 3$

×

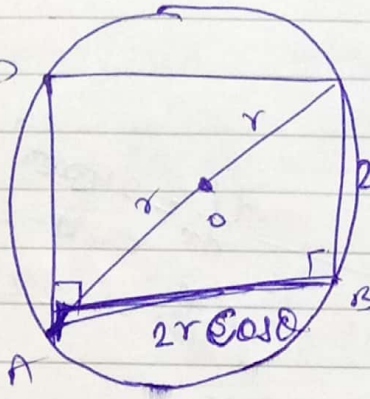
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2 days off. (27,28)

$$\left(\frac{d^2v}{dx^2}\right)_{x=3} < 0$$

Volume is maximum at $x = 3$ cm.

Q.19



Given → r

$z = AB \times BC$

$z = 2r\cos\theta \cdot 2r\sin\theta$

$z = 2r^2 \sin 2\theta$

$$\frac{dz}{d\theta} = 4r^2\cos 2\theta = 0$$

$\cos 2\theta = 0 = \cos \pi/2$

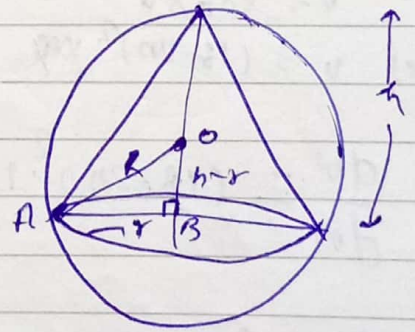
$2\theta = \pi/4$

★★★★

Seen Mid = 15

Q.23

Given ⇒ sphere of radius R



(Max) $V = \frac{1}{3}\pi r^2 h$

$V_c = \frac{8}{27} V_s$

In $\triangle ABO$

$R^2 = h^2 + R^2 - 2hr + r^2$

$r^2 = 2hr - h^2$

$V = \frac{\pi}{3}(2hR - h^2) \times h$

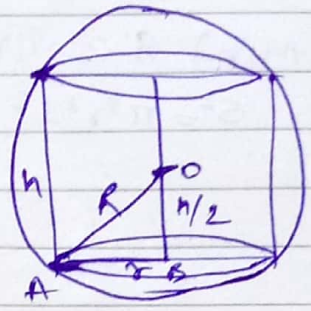
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4.

$V = \frac{\pi}{3}(2h^2R - h^3)$ ← second order

Misc

Q.17



We know that Given (a)

$V_{cl.} = \pi r^2 h$

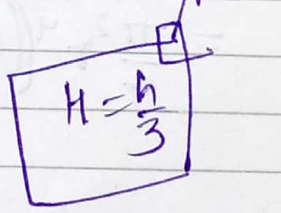
In $\triangle ABO$

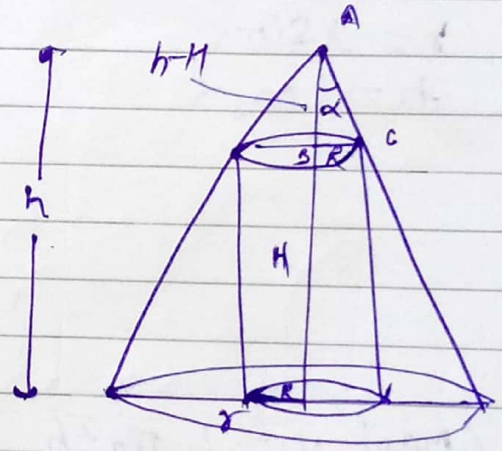
$R^2 = \frac{h^2}{4} + r^2$

$r^2 = R^2 - \frac{h^2}{4}$

$V = \pi\left(R^2 - \frac{h^2}{4}\right) \times h$

$\frac{dV}{dh} =$





$V_{cl.} = \pi R^2 H$

$V_{cl.} = \pi R^2 (h + R\cot\alpha)$

$\tan\alpha = \frac{R}{h-H}$

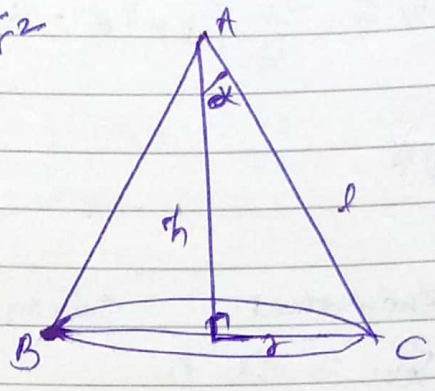
$R\cot\alpha = h - H$

$H = h - R\cot\alpha$

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Ex 6.5
Q.24 Prove

Given $V = \frac{1}{3}\pi r^2 h \Rightarrow h = \frac{3V}{\pi r^2}$



(Min) $S = \pi r l$; $h = \sqrt{2}r$

$S^2 = \pi^2 r^2 l^2 \Rightarrow \pi^2 r^2 (h^2 + r^2)$

$\Rightarrow \pi^2 r^2 \left(\frac{9V^2}{\pi^2 r^4} + r^2\right) = \frac{9V^2}{r^2} + \pi^2 r^4$

$\frac{d^2}{dr} = \left[9V^2 \times \frac{-2}{r^3} + \pi^2 4 r^3\right]$

Prove

(25) Given l

(Max) $V = \frac{1}{3}\pi r^2 h$

$\alpha = \tan^{-1}\sqrt{2}$

$r = l \sin\alpha$

$h = l \cos\alpha$

Given $S = \pi r l + \pi r^2$

$= \frac{\pi}{9} 2 \times r^4 \times (l^2 - r^2)$

$= \frac{\pi}{r} 2 r^4 \left[\left(\frac{S - \pi r^2}{\pi r}\right)^2 - r^2\right]$

Prove

(26) (Max) $V = \frac{1}{3}\pi r^2 h$

$= \frac{\pi}{9} 2 \times r^4 \times (l^2 - r^2)$.

$\alpha = \sin^{-1}\frac{1}{3}$

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1Q - 6 Mark

chapter - 1

Relation & function

Relation :

* Cartisian Product : ⇒ Given two non-empty set A & B. The cartesian Product A×B is the set of all ordered pair of the form (a, b) where the first entry comes from set A & second comes from set B.)

(1) $A = \{1, 2\}$ $B = \{P, Q\}$

$A \times B = \{(1,P)(1,Q)(2,P)(2,Q)\}$

Cartisian Product

* Relation :

Let A and B be two sets. then a relation R from A to B is a subset of A×B thus R is a Relation A to B ⇒ $R \subseteq A \times B$

$R \subseteq A \times B$

$R_1 = \{(1,P)(Q,2)\}$. $R_1 \nsubseteq A \times B$

$R_2 = \{(1,P)(2,Q)(1,Q)\}$ $R_2 \subseteq A \times B$

Domain : $\{1, 2\}$

$R = \{P, Q\}$.

* Type of Relation :

(1) Reflexive : $x \, R \, x$, $\forall \, x \in$ set

$A = \{1, 2, 3, 4\}$

$R = \{(1,1)(1,2)(1,3)\}$

(2) Symmetric : If $x \, R \, y \Rightarrow y \, R \, x$, $\forall \, x, y \in$ set

$A = \{1, 2, 3, 4\}$

$R = (1,1)(2,2)(1,2)(2,1)$

(3) Transitive : If $x \, R \, y$ and $y \, R \, z \Rightarrow x \, R \, z \Rightarrow \forall \, x, y, z \in$ Set

$A = \{1, 2, 3, 4\}$

$R = (1,2)(2,1)(1,1)$

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(4) Equivalence : (1) + (2) + (3) ✓

Ex: $A = \{1,2,3\}$

$R = \{(1,1)\ (2,2)\ (3,3), (1,2)\ (2,1)\}$

Q.1 $R \to N$

(ii) $R = \{(x,y) : y = x+5 \text{ and } x<4\}$

$R = \{(1,6)\ (2,7)\ (3,8)$

Ro - Transitive $(1,6)\ (6, \overset{X}{-})$

$(2,7)\ (7, \overset{b}{-})$

$(3,6)\ (8, \overset{X}{-})$

Q.1 (iv) $R \to Z$

$R = \{(x,y) : x-y \text{ is an integer}\}$

$xRy \Rightarrow x-y = k \in I$

i) If reflexive $xRx \ \forall \ x \in Z$

$x - x = 0 \in I$

$\Rightarrow$ R is Reflexive.

(2) symmetric By def. If $x,y \Rightarrow yRx \ \forall \ x,y \in Z$

$\Rightarrow -(y-x) = k \in I$

$\Rightarrow y - x = -k \in I$

$\Rightarrow yRx \Rightarrow R$ is symmetric

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③ Transitive : By def.

If xRy and $yRz \Rightarrow xRz \ \forall \ x, y, z \in Z$

$x R y \Rightarrow x - y = k_1 \in I \quad - (1)$

$yRz \Rightarrow y - z = k_2 \in I \quad - (2)$

from eq (1) + (2)

$(x-y) + (y-z) = k_1 + k_2$

$(x-z) = (k_1 + k_2) \in Z \Rightarrow xRR \Rightarrow R$ is a transitive

① + ② ✓ ⇒ True.

R is an equivalence

Q.a $R = \{(a,b) : |a-b| \text{ is even no.}\}$

$A = \{1, 2, 3, 4, 5\}$.

$aRb \in |a-b| = 2k$

(1) Ref. - By def If $aRa \ \forall \ a \in A$

$\Rightarrow |a-a| = 0$ is a even no.

$\Rightarrow R$ (ref.)

(2) Sym - By def If $aRb \Rightarrow bRa \ \forall a, b \in A$

$aRb \Rightarrow |a-b| = 2k$

$\Rightarrow |b-a| = 2k$

$\Rightarrow bRa \Rightarrow R$ is Sym.

(3) Transitive. ⇒ By def.

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aRb and $bRc \Rightarrow aRc \quad \forall a, b, c \in A$

$(aRb) \Rightarrow |a-b| = 2k_1$

$\Rightarrow (a-b) = \pm 2k_1 \quad \text{---} \quad (1)$

$bRc \Rightarrow |b-c| = 2k_2$

$\Rightarrow (b-c) = \pm 2k_2 \quad \text{---} \quad (2)$

By adding (1) and (2)

$|a-c| = 2(\pm k_1 \pm k_2)$

$aRc \Rightarrow R$ is trans.

HP2+3 True R is Equi.

Q.9

$A = \{0, 1, 2, \ldots\}$

$aRb = \{|a-b| = 4k, k \in I\}$

$xRy \Rightarrow |x-1| = 0, 4, 8, 12$

$x-1 = 0 \Rightarrow x = 1$

$x-1 = 4 \Rightarrow x = 5$

$x-1 = 8 \Rightarrow x = 9$

$x-1 = 12 \Rightarrow x = 13 \notin A$

Equi. class of $1 = [1] = \{1, 5, 9\}$.

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Q.2

$aRb \Rightarrow a \leq b^2 \quad a, b \in R$

(1) Ref. $aRa \quad \forall a \in R$

$\therefore a = \frac{1}{2} \Rightarrow \frac{1}{2} \leq \frac{1}{4}$ [Not possible]

R is not ref.

(2) Sym: If $aRb \Rightarrow bRa \; \forall a, b \in R$

Let $a = 1, b = 2$

$aRb \Rightarrow a \leq b^2 \Rightarrow 1 \leq 2^2 \Rightarrow 1 \leq 4$

$bRa = b \leq a^2 \Rightarrow 2 \leq 1$ [Not Possible]

$\Rightarrow b \not R a \Rightarrow R$ is not sym-sym

(3) Trans

Let $a = 2, b = -3, c = 1$

$aRb \Rightarrow 2 \leq (-3)^2 = 2 \leq 9$

$bRc \Rightarrow -3 \leq 1^2 \Rightarrow -3 \leq 1$

$aRc \Rightarrow 2 \leq 1^2 \Rightarrow 2 \leq 1$ [Not Possible]

R is not trans.

Q.5

$aRb \Rightarrow a \leq b^3$

- Ref - $a = 1/2$
- Symm - $a = 1, b = 2$
- Trans - $a = 100, b = 5, c = 2$

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Q.4

$aRb \Rightarrow a \leq b$

ref.

$aRa \Rightarrow a \leq a$ ✓

② Sym: $a=1, b=2$

⑤ Trans:

$aRb \Rightarrow a \leq b$ — ①

$bRc = b \leq c$ — ②

$\Rightarrow a \leq c$

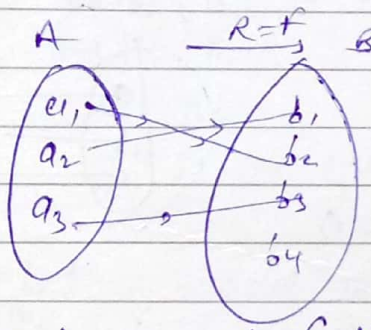
$\Rightarrow aRc$

R is transitive.

✱ Exercise - 1.2

Function: If every element of set A is uniquely related to set B and it is denoted by $f: A \to B$ or $A \xrightarrow{f} B$

(2)



$f: A \to B$

A: Domain, B: range

Domain: set (Value of x ∈ R where f^n defined)

$\pm \sqrt{(\ )}$, $\frac{1}{0}$, x^0.

Codomain: Set B.

A{1,2,3}
B{1,4,9,16}

Range: ① finite → value of f^n ∈ R

② Infinite: Domain of f^{-1} = Range of f {1,4,9}

③ Range → set of Image {1,4,9}

④ Preimage = {1,2,3}

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* Type of f^n : [Do not use graph]

(1) one-one (Injective) mapping : 1-1 $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

(2) many-one (not 1-1) 2-1 $f(x_1) = f(x_2) \Rightarrow x_1 \neq x_2$

(3) onto / (Surjective) : Range = Codomain → Every element of Codomain has Pre-Images in domain

(4) into Range ≠ Codomain → Every element of Codomain do not have Pre Image in domain

(5) Bijective ⇒ one-one + onto

~~(1) one-one (Injective) :~~

~~(I) one (Injective) :~~

$f : N \to N$

Q $f(x) = x^2$

one-one

A f^n is said to be one-one f^n or injective mapping if different element of A have different image in B.

(i) for one-one / many one :

We have $f(x_1) = f(x_2) \quad \forall\ x_1, x_2 \in N$

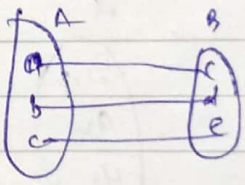
many one

* f^n is said to be many one f^n if two or more element of A have the same image in B.

$\Rightarrow x_1^2 = x_2^2$ ($x_1 \neq x_2$)

$x_1^2 - x_2^2 = 0$

$(x_1 - x_2)(x_1 + x_2) = 0$



$f(x_1) = f(x_2)$ $\quad x_1 + x_2 \neq 0$ $\quad x_1 - x_2 = 0$ one-one

$x_1 \neq x_2$ $(x_1, x_2 \in N)$ $\quad x_1 = x_2$

if $f(x_1) \neq f(x_2) \Rightarrow x_1 \neq x_2$

(2) onto / Onto :

let $y = f(x)$

$y = x^2$

$x = \sqrt{y} \quad \forall\ y \in N$ (Codomain)

$x \notin N$ (domain)

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onto ⇒ If every element of B is the image of some element of A under f.



$\therefore$ Range $\neq$ codomain

Hence

f is into.

into ⇒ if at least one element in co-domain which is not the image of any element in domain



Example 12 (Very Imp)

$$f: N \to$$

$$f(n) = \begin{cases} n+1 & ; n = \text{odd} \\ n-1 & ; n = \text{even} \end{cases}$$

(i) For one-one

C-1 Let n_1, n_2 being even

$$f(n_1) = f(n_2)$$

$$n_1 - 1 = n_2 - 1$$

$$n_1 = n_2$$

f is one-one

(ii) Let n_1, n_2 being odd no

$$f(n_1) = f(n_2)$$

$$n_1 + 1 = n_2 + 1$$

$$n_1 = n_2$$

f is one-one

C-3 Let n_1 = odd and n_2 = even

$$f(n_1) = f(n_2)$$

$$n_1 + 1 = n_2 - 1$$

$$n_2 - n_1 = 2 \quad \text{(Not Poss.)}$$

$$n \neq n_2 \Rightarrow f(n_1) \neq f(n_2)$$

f is one one

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(2) for onto

(i) Let $n = n$(odd) : $n = n+1$ (even)

$f(n+1) = (n+1)-1 = n_{odd}$

(2) Let $x = n$(even) ; $n = n-1$ (odd)

$f(x-1) = n-1+1 = n$

(even)

Range = Codomain

f^n is onto

Q.6

$f : R_+ \to [-4, \infty)$

$f(n) = x^2 + 4$

Ex-1.3 Inverted :

A f^n $f(x)$ is invertable $\Rightarrow$ f^n is bijective (one-one, onto)

First Prove (one one, nto)

(1) for one one

Let $n_1, n_2 \in R_1$

$f(n_1) = f(n_2) \quad x = x_2$

Prove $n_1^2 + 4 = n_2^2 + 4$

$(n_1 + n_2)(n_1 - n_2) = 0$

$(n_1 + n_2 \neq 0) : x_1 = x_2$

$f(n_1) \neq f(n_2) \quad x_1 \neq x_2$

$n_1, n_2 \in R_1$

(2) for onto

Let $y = f(n)$

$y = n^2 + 4$

$n = \sqrt{y-4} \geq 0$

$y - 4 \geq 0$

$y \geq 0$

Range $= [4, \infty)$ = Codomain

f^n is onto

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(3) Inverse

Let $y = f(x) \Rightarrow x = f^{-1}(y)$

$y = x^2 + 4$

$x = \sqrt{y-4}$ $\quad f^{-1}(y) = \sqrt{y-4}$

$f^{-1}(x) = \sqrt{x-4}$.

Q.9

$f: R_+ \to [-5, \infty)$

$f(x) = 9x^2 + 6x - 5$

for one one

Let $x_1, x_2 \in R_+$

$f(x_1) = f(x_2)$

$9x_1^2 + 6x_1 - 5 = 9x_2^2 + 6x_2^2 - 5$

$9(x_1 + x_2)(x_1 - x_2) + 6(x_1 - x_2) = 0$

$(x_1 - x_2)[9(x_1 + x_2) + 6] = 0$

$x_1 = x_2$

for onto

Let $y = f(x)$

$y = 9x^2 + 6x - 5$ $\quad ax^2 + 6x - 5 - y = 0$

$a = 9, \; b = 6 \quad c = -5 - y$

$x = \frac{-6 \pm \sqrt{36 + 4 \times 9 \times (5+y)}}{2 \times 9}$

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$$x = \frac{3(-1 \pm \sqrt{6+y})}{2 \times 3}$$

$x = \frac{-1+\sqrt{6+y}}{3} \in R$, and $x = \frac{-1-\sqrt{6+y}}{3} \notin R$,

$\frac{-1+\sqrt{6+y}}{3} \geq 0$

$\sqrt{6+y} \geq 1$

$y \geq -5 \Rightarrow [-5, \infty) = R$ = Codomain = onto

Ex-1.4

* Binary operations :

3

$* : A \times A \to A$

$f : A \to B$

* binary operation $(*)$ on the element a and b of a set A is a function $*$ $A \times A \to$ and denoted $a * b$.

Property :

(1) Comm : $a * b = b * a$

(2) Asso : $a * (b * c) = (a * b) * c$

(3) Identity : $a * e = a = e * a$

(4) Inverse : $a * a^{-1} = e = a^{-1} * a$

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Q. $a*b=(a+b)+(ab) \quad \forall\, a,b \in R-\{-1\}$

(1) Comm! $\rightarrow a*b = b*a \quad \forall\, a,b \in R-\{-1\}$

LHS $= a*b = (a+b)+(ab)$

RHS $= b*a = (b+a)+(ba)$

By comm. Rule on $+$ and $\times$

$a*b = b*a$

$*$ is a comm.

(2) Asso: by def.

$(a*b)*c = a*(b*c) \quad \forall\, a,b,c \in R-\{-1\}$

LHS $\Rightarrow (a*b)*c$

$\Rightarrow (a+b+ab)*c$

$(a+b+ab+c)+(a+b+ab).c$

$a+b+c+ab+bc+ca+abc$

RHS $\Rightarrow a*(b*c)$

$a*(b+c+bc)$

$(a+b+c+bc)+a\times(b+c+bc)$

$a+b+c+bc+ab+ac+abc.$

(3) For Identity element:

$a*e = a = e*a \quad \forall\, a,e \in R-\{-1\}$

$(a+e+ae) = a$

$e(1+a) = 0$

$1+a \neq 0 \quad a \in R-\{-1\}$

$e = 0$

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For inverse:

By defination

$$a * a^{-1} = e = a^{-1} * a \qquad \forall\ a, e \in R - \{-1\}$$

$$a + a^{-1} + a \cdot a^{-1} = 0$$

$$a^{-1} = \frac{-a}{1+a} \in R - \{-1\}$$

Que: Let $*$ be a
if $a * b = 2a + b - 3$
find $3 * 4$
$\Rightarrow 2(3) + 4 - 3 = 7$ Ans

Que: (11)

$$(a,b) * (c,d) = (a+c,\ b+d)$$

(1) Comm.: $A * B = (a,b) * (c,d) = (a+c,\ b+d)$

$B * A = (c,d) * (a,b) = (c+a,\ d+b)$

Important Questions

(2) Assoc: $(A * B) * C$

$$[(a,b) * (c,d)] * (e,f)$$

$$(a+c,\ b+d) * (e,f) = (a+c+e,\ b+d+f)$$

(3) Identity

$e = (e_1, e_2) \in (N \times N)$

$a + e = a$

$(a,b) * (e_1, e_2) = (a,b)$

$(a + e_1,\ b + e_2) = (a,b)$ $\quad a + e_1 = 0 \Rightarrow a = e_1 = 0$

$b + e_2 = b = e_2 = 0$

$(e_1, e_2) = (0,0) \notin N \times N$

(4) Inverse: $a * a^{-1} = e$

$(a \times b) * (a,y) = (0,0)$

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$x = -a, \quad y = -b$

$a^{-1} = (x, y) = (-a, -b) \in \mathbb{R} \times \mathbb{R}$

Miscellaneous

Q.11 $a * b = \begin{cases} a+b & ; a+b < 6 \\ a+b-6 & ; a+b \geq 6 \end{cases}$

$e = 0, \quad a^{-1} = 6 - a$

$a \neq 0$

for e

If $a+b < 0$

$a * b = a + b$

By def: $a * e = a$

$a + e = a$

$e = 0$

If $a+b \geq 6$

$a * e = a$

$a + e - 6 = a$

$e = 6 \notin S$

(2) for a^{-1}

If $a+b < 6$

By $a * a^{-1} = e$

$a + a^{-1} = 0$

$a^{-1} = -a \notin S$

$a * b = a + b - 6$

If $a+b \geq 6$

$a * a^{-1} = e$

$= 0$

$a + a^{-1} - 6 = 0$

$a + a^{-1} = 6$

$a^{-1} = 6 - a \quad a \neq 0$

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02/01/18 Ch-2 (4 Mark)

ITF (Inverse trigonometric function)

Inverse function which is related to trigonometric ratio be called of inverse trigonometric function.

If $\sin\theta = x$

then $\theta = \sin^{-1}x$

$\theta \in [-\pi/2, \pi/2]$

$x \in [-1, 1]$

(1) $\sin(\sin^{-1}x) = x \in [-1,1]$

$\sin^{-1}(\sin x) = x \in [-\pi/2, \pi/2]$

(2) $\sin^{-1}(1/x) = \operatorname{Cosec}^{-1}x$

$\cos^{-1}(1/x) = \sec^{-1}x$

$\tan^{-1}(1/x) = \cot^{-1}x$

(3)
$$\begin{cases} \sin^{-1}(-x) = -\sin^{-1}x \\ \tan^{-1}(-x) = -\tan^{-1}x \\ \operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1}x \end{cases}$$

$$\begin{cases} \cos^{-1}(-x) = \pi - \cos^{-1}x \\ \cot^{-1}(-x) = \pi - \cot^{-1}x \\ \sec^{-1}(-x) = \pi - \sec^{-1}x \end{cases}$$

cosec
cos sec cot

(4) $\sin^{-1}x + \cos^{-1}x = \pi/2$

$\tan^{-1}x + \cot^{-1}x = \pi/2$

$\sec^{-1}x + \operatorname{cosec}^{-1}x = \pi/2$

$= \frac{\pi}{2}$

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$\pi - \frac{\pi}{3}$ $\pi - \frac{\pi}{4}$

(1)

* (1) $2\sin^{-1}x = \sin^{-1}[2x\sqrt{1-x^2}] = \sin^{-1}(2\sin\theta\cos\theta)$

Proof $x = \sin\theta$

$\theta = \sin^{-1}x$

RHS $\sin^{-1}[2\sin\theta\cos\theta]$

$= \sin^{-1}[\sin 2\theta)$

$= 2\theta$

$= 2\times\sin^{-1}x.$

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(2) $2\cos^{-1}x = \cos^{-1}[2x^2-1]$

(3) $2\tan^{-1}x = \tan^{-1}\left[\frac{2x}{1+x^2}\right] = \sin^{-1}\left[\frac{2x}{1+x^2}\right]$

$= \cos^{-1}\left[\frac{1-x^2}{1+x^2}\right]$

(6)

* $2\sin^{-1}x = \sin^{-1}(3x-4x^3)$

$3\cos^{-1}x = \cos^{-1}(4x^3-3x)$

$3\tan^{-1}x = \tan^{-1}\left[\frac{3x-x^3}{1-3x^2}\right]$

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* $\tan^{-1}x \pm \tan^{-1}y = \tan^{-1}\left[\frac{x \pm y}{1 \mp xy}\right]$

Q. $\sin^{-1}\left(\sin\frac{2\pi}{3}\right) \Rightarrow \sin^{-1}\left[\sin\left(\pi-\frac{\pi}{3}\right)\right] = \sin^{-1}\left(\sin\frac{\pi}{3}\right]$

$= \frac{\pi}{3} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(2) $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$

$\frac{3\pi}{4} = \pi - \frac{\pi}{4}$

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(3) $\sin^{-1}\left(\sin\frac{3\pi}{5}\right)$

$\frac{3\pi}{5} = \pi - \frac{2\pi}{5}$

(4) $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$

$\pi + \frac{\pi}{6}$

(5) $\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$

$\left(2\pi + \frac{\pi}{6}\right)$

Q.14 $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$

$\tan^{-1}\left(\tan\frac{\pi}{3}\right) - \left[\pi - \sec^{-1}(2)\right]$

$\frac{\pi}{3} - \left[\pi - \sec^{-1}\left(\sec\frac{\pi}{3}\right)\right]$

$\frac{\pi}{3} - \left[\pi - \frac{\pi}{3}\right]$

Q.5 $\tan^{-1}\left[\frac{\sqrt{1+x^2}-1}{x}\right]$

$x = \tan\theta \quad \therefore \theta = \tan^{-1}x$

$\tan^{-1}\left[\frac{\sec\theta - 1}{\tan\theta}\right] = \tan^{-1}\left[\frac{1-\cos\theta}{\sin\theta}\right]$

$\tan^{-1}\left[\frac{2\sin^2\theta/2}{2\sin\theta/2\cos\theta/2}\right] = \tan^{-1}(\tan\theta/2) = \theta/2$

$= \frac{1}{2}\tan^{-1}x$

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full example.

Q. $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$

$\tan^{-1}\left(\frac{x-1}{x-2}\right) = \tan^{-1}(1) - \tan^{-1}\left(\frac{x+1}{x+2}\right)$

$= \tan^{-1}x - \tan^{-1}y$ (use formula)

Misc.

Q.9 $\frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\cos^{-1}(\cos 2\theta)$

Put $x = \tan^2\theta$

$\sqrt{x} = \tan\theta$

$\theta = \tan^{-1}\sqrt{x}$

$= \frac{1}{2} 2\theta =$

$= \theta$

$= \tan^{-1}\sqrt{x}.$

(2)

Q.11 $\tan^{-1}\left[\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}\right] = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x.$

$x = \cos 2\theta$

$\theta = \frac{1}{2}\cos^{-1}x$

$\tan^{-1}\left[\frac{\sqrt{2}\cos\theta - \sqrt{2}\sin\theta}{\sqrt{2}\cos\theta + \sqrt{2}\sin\theta}\right]$

$\tan^{-1}\left[\frac{1 - \tan\theta}{1 + \tan\theta}\right]$

$\tan^{-1}\left\{\tan\left(\frac{\pi}{4} - \theta\right)\right\}$

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$\cos 2\theta = 1 + 2\sin^2\theta$

Example Miss.. Due

Example - 12, 13

* $\sqrt{(1+\sin 2x)} = \cos x + \sin x$ Example

Q.10

$$\cot^{-1}\left[\frac{(\cos x/2 + \sin x/2) - (\cos x/2 - \sin x/2)}{(\cos x/2 + \sin x/2) + (\cos x/2 - \sin x/2)}\right]$$

$$\cot^{-1}\left[\frac{2\sin x/2}{2\cos x/2}\right] = \cot^{-1}[\tan x/2]$$

$$= \cot^{-1}\left[\cot\left(\frac{\pi}{2} - \frac{x}{2}\right)\right]$$

Q13

$$2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$$

$$2\tan^{-1}\left[\frac{2\cos x}{\sin^2 x}\right] = \tan^{-1}\left[\frac{2}{\sin x}\right]$$

$$x = \pi/4$$

(16) Solve $\sin^{-1}(1-x) - 2\sin^{-1}x = \pi/2$

Easy

easy easy

$$\sin^{-1}(1-x) = \frac{\pi}{2} + 2\sin^{-1}x$$

$$1-x = \sin\left(\frac{\pi}{2} + 2\sin^{-1}x\right)$$

$$1-x = \cos(2\sin^{-1}x)$$

$$1-x = 1 - 2[\sin(\sin^{-1}x)]^2$$

$$1-x = 1-2x^2$$

$$2x^2 - x = 0$$

$$x(2x-1) = 0$$

$x = 0$ ✓ $\quad x = 1/2$ ✗

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